Strings, gauge theory and gravity

Storrs, Feb. ‘07
Outline

- Motivation - why study quantum gravity?
- Intro to strings
- Gravity / gauge theory duality
- Gravity $\Rightarrow$ gauge: Wilson lines, confinement
- Gauge $\Rightarrow$ gravity: black holes from gauge theory
Motivation

Good theories often carry the seeds of their own destruction. For example blackbody radiation can’t be understood using classical E&M.

But the standard model of particle physics is a remarkably robust theory. It could well be valid to extremely high energies.
For example, consider dialing the one parameter in the standard model that hasn’t been directly observed: the Higgs mass $M_h$. As a function of $M_h$ you can plot the energy scale $\Lambda$ where the standard model breaks down.
From Dawson, hep-ph/9901280:

![Graph showing Landau pole and potential bounded from below.](image)

- **Landau pole**
- **Potential bounded from below**
- **Allowed region**

**LEP limits:**

$$114 \text{ GeV} < m_h < 219 \text{ GeV}$$

**Scale of new physics**

$$\Lambda (\text{GeV}) = \text{scale of new physics}$$
So we don’t get much guidance from staring at the standard model. What’s a poor theorist to do?

- Experimental input: neutrino masses, dark matter, accelerated expansion, ...
- Theoretical puzzles: hierarchy problems (explain electroweak scale, vacuum energy, ...)

but the one I’d like to focus on is

- Theoretical problem: the one force that’s left out of the standard model – gravity – doesn’t play by the usual rules of the game.
The problem is that gravity becomes strong at high energies.

\[ F = G \frac{m_1 m_2}{r^2} \implies F = G \frac{E_1 E_2}{r^2 c^4} \]

This makes it hard to construct a theory of quantum gravity (virtual particles can have arbitrarily large energies).

Why worry, since classical GR describes the world so well?

Singularity theorems \(\implies\) short distance / high energy phenomena are unavoidable (big bang, black holes).
The universe had to begin in a state where the short distance behavior of gravity was important. Besides, right in our neighborhood

\[
4 \times 10^6 \, M_{\odot} \text{ in } 1''
\]

galactic center

We need a well-behaved quantum theory of gravity!
Intro to strings

The best-developed approach to quantum gravity is string theory. Based on a very simple idea: replace point particles with strings.

- open string
- closed string
Compare the first excited state of an open string

to an electromagnetic wave

Both carry spin 1, and both are massless (in 26 dimensions).

=> open strings incorporate gauge fields!
Compare the first excited state of a closed string to a gravity wave.

Both carry spin 2, and both are massless (in 26 dimensions).

=>$\text{closed strings incorporate gravity!}$
You get the right types of interactions, from strings joining and splitting.

This could describe a photon emitting a graviton.
So string theory is automatically a theory of quantum gravity, and it has all the ingredients one needs to build models of particle physics (gauge fields, fermions,...).

Main topic:

It leads to surprising connections between gravity and gauge theory.
Gravity / gauge duality

Open strings can be restricted to end on “D-branes.”

This gives rise to

- a gauge theory in p+1 dimensions
- gravity in all dimensions
Usually both open and closed strings are important. But in a special “decoupling limit” one can have a complete description either in terms of open strings, or in terms of closed strings.

Decoupling?
Go to low energies => only massless open strings important, but gravitational redshift => closed strings survive.

The two descriptions are equivalent. This leads to gravity / gauge duality.
Prototype example: (Maldacena)

Gravity (closed string theory) on $\text{AdS}_5 \times S^5$ is equivalent to a gauge theory ($\mathcal{N} = 4$ susy Yang-Mills) on the boundary of $\text{AdS}$.

Can take this as the definition of AdS supergravity.
Gravity $\Rightarrow$ gauge theory

An interesting quantity in the gauge theory is the interquark potential, or equivalently the Wilson loop expectation value.

\[
\langle 0 | \text{Tr} P e^{i \oint A_\mu dx^\mu} | 0 \rangle
\]

Need to compute

\[
V(r) = ?
\]
In the gauge theory this is a hard calculation. One has to study the electric field.

![Diagram](image)

- gauge theory spacetime (boundary of AdS)

Non-abelian theory so the flux lines self-interact.
How does it look in supergravity? The quark and antiquark are connected by a string which sags towards the center of AdS.

You can easily compute the energy of the string.

\[ V(r) = -\frac{4\pi^2 \sqrt{2g_{YM}^2 N}}{\Gamma(1/4)^4 r} \]  

(Maldacena, Rey & Yee)
Cute extension: you can put a D-brane in AdS, so the strings don’t have to end on the boundary.

This gives the quarks a finite mass. You can compute the potential $V(m, r)$. (DK & Lifschytz)

$$r(m, z) = \frac{1}{\pi m} \left( \frac{g_{YM}^2 N}{2} \right)^{1/2} z^{1/2}(z + 1)^{1/4} F\left(\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, -z\right)$$
How would things look in a confining theory like QCD? The potential well would cut off at a finite depth. Finite redshift $\Rightarrow$ finite energy / unit length for a string parallel to the boundary.

The hard part is understanding the geometry near the boundary – it isn’t AdS any more. (Witten,...)
Gauge theory $\Rightarrow$ gravity

Gravity / gauge duality gives a new perspective on long-standing problems of quantum gravity.

For example, can we understand black holes from the gauge theory perspective?

Study the duality at finite temperature.
gravity description: get a black hole in AdS

gauge theory description: thermal plasma

black hole \approx \text{thermal gauge theory}
What does absorption by a black hole look like?

**gravity description:**
- object falls towards center of AdS,
- trapped once it crosses the horizon
- absorption time $\tau_{\text{abs}}$

**gauge theory description:**
- approach to thermal equilibrium
- equilibration time $\tau_{\text{equil}}$

There's a simple model of the gauge theory in which the timescales agree.  

Iizuka, DK, Lifschytz, Lowe

What happens inside the horizon?
The region inside the horizon can’t be seen from the boundary - the light cones tip over.

So how can it be described in the gauge theory?
Hamilton, DK, Lifschytz, Lowe:

Local operators in the bulk of AdS correspond to non-local operators on the complexified boundary.

For example in $\text{AdS}_3$

$\phi(T, X, Z)$ local bulk operator (massless scalar)

$\mathcal{O}(T, X)$ local boundary operator (dimension 2)

$$\phi(T, X, Z) \Leftrightarrow \frac{1}{\pi} \int_R dT' dY' \mathcal{O}(T + T', X + iY')$$

$R = \{T'^2 + Y'^2 < Z^2\}$

radial coordinate in AdS

complex spatial coordinates on bdy
Why?

\[ ds^2_{AdS} = \frac{1}{Z^2} \left( -dT^2 + dX^2 + dZ^2 \right) \]

Wick rotate \( X = iY \)

\[ \Rightarrow \quad ds^2 = \frac{1}{Z^2} \left( -dT^2 - dY^2 + dZ^2 \right) \]

This is de Sitter space (\( Z = \) time coordinate).

Conventional initial value problem in de Sitter space.
This construction applies directly to $\text{BTZ} = \text{AdS}_3/\mathbb{Z}$.

You can use it to represent local operators inside the BTZ horizon, or even at the BTZ singularity.

An operator at the singularity corresponds to a non-normalizeable operator in the boundary theory, with divergent correlation functions.
Conclusions

Gravity / gauge duality is a powerful tool for studying gauge theory, and provides a new perspective on quantum gravity.

So far, it has only been precisely formulated in anti-de Sitter space.

Q. Is there a dual gauge-theory-like description of a realistic cosmology?