CURRENT RESEARCH

NON-LOCAL OPERATORS IN THE AdS/CFT CORRESPONDENCE

In the context of the AdS/CFT correspondence it is well-known that correlation functions of local operators in the CFT are dual to boundary correlators computed in anti-de Sitter space. But from the point of view of AdS supergravity one of the most interesting questions is how quasi-local bulk gravitational physics emerges from the CFT. The most direct way to address this question is to study correlators of non-local operators in the CFT.

At leading order in the $1/N$ expansion a local bulk quantum field has been expressed as a non-local boundary operator in [1]. Together with Gilad Lifschytz, David Lowe and Alex Hamilton I am attempting to extend this work in several directions. In [2] we constructed non-local operators in conformal quantum mechanics that are dual to local bulk operators in AdS$_2$. We showed that the boundary operator could be localized at spacelike separation from the bulk point. This provided a precise signal for bulk phenomena such as the horizon of the Poincaré patch or the Rindler horizon of an AdS$_2$ black hole. We also showed that points inside the horizon of an AdS$_2$ black hole are encoded in the boundary theory, but only in terms of operators that act on both copies of the thermofield doubled CFT. We are currently working to extend these results to AdS$_3$. We then hope to apply them to study the singularity of a BTZ black hole.

FLUX COMPACTIFICATIONS OF STRING- AND M-THEORY

Recently there has been a great deal of interest in constructing generic string vacua with $\mathcal{N} = 1$ supersymmetry. A large class of such vacua can be obtained by introducing magnetic fluxes on the compactification manifold [3]. These vacua have few or no moduli, and may generate a positive cosmological constant after supersymmetry breaking [4].

It would be extremely interesting to understand the general structure of $\mathcal{N} = 1$ vacua. However the original work [3] assumed that the compactification
manifold admitted a Killing spinor with definite chirality. Bobby Acharya and I are studying what happens if we relax this assumption, along the lines of [5]. Within a specific ansatz we hope to explicitly construct more general flux vacua of M-theory; these vacua should arise from compactification manifolds with intrinsic torsion [6].

**Particle and string production in time-dependent backgrounds**

String theory in time-dependent backgrounds is very poorly understood. Partly for this reason string theory has had very little to say about cosmology. Recently it was pointed out that simple solutions to the vacuum Einstein equations in higher dimensions, involving warped compactification on a hyperbolic manifold $H$, give rise to 3+1 dimensional FRW cosmologies with several quasi-realistic features: an initial big bang singularity, followed by a period of inflation, followed by decelerated expansion at late times [8].

Hamilton, Iizuka, Parikh and I are studying perturbative string theory in this background. By construction the background satisfies the leading $O(\alpha')$ equations of motion. Moreover there is a reliable $\alpha'$ expansion, in powers of $\alpha'/(\text{radius of } H)^2$. We hope to use this expansion to systematically compute string production during the period of inflation. As an intermediate step, we have reformulated particle production in this background in terms of a single-particle path integral [9]. This provides a novel first-quantized alternative to the standard Bogolubov calculation. As an added bonus certain properties, such as the dependence of particle production on the initial state of the field, are much easier to understand in first-quantized language.

**Global structure of metastable de Sitter vacua**

de Sitter space has been argued to be unstable. If one waits long enough it will decay back to flat space. This raises an interesting question: what is the global structure of such a metastable de Sitter universe? Gao and Wald have shown that under certain conditions a perturbed de Sitter universe is “tall,” with no particle horizons at late times [10]. Susskind and I are studying ways of extending this result to the more interesting case of a metastable de Sitter universe which is locally asymptotically Minkowski both in the far past and future.
Recently completed projects

Brane gas cosmology in M-theory

Some time ago Brandenberger and Vafa made an intriguing proposal for early-universe cosmology in string theory [11]. Their scenario is that the universe started out in a small hot dense state, with large numbers of strings wound on non-contractable cycles. Eventually a thermal fluctuation makes $d$ of the spatial dimensions expand. If $d \leq 3$ the strings wound on those dimensions can intersect and annihilate, enabling decompactification.

This may provide a natural mechanism for decompactifying three spatial dimensions in string theory. It is not obvious, however, that the mechanism works at the non-perturbative level. The Brandenberger-Vafa scenario seems to rely on having one-dimensional extended objects. It also seems to require dilaton gravity. But these aspects of string theory are both absent in M-theory.

To investigate this issue Easther, Greene, Jackson and I formulated a version of the Brandenberger-Vafa scenario in M-theory compactified on $T^{10}$ [12, 13]. We took the torus to be spatially homogeneous, characterized by ten independent radii, and introduced a gas of supergravity particles and wrapped M2-branes. To satisfy Gauss’ law equal numbers of branes and antibranes must be wrapped on any given 2-cycle. Several new results were obtained in this framework.

Late-time attractor solutions

At early times the brane gas is in thermal equilibrium, but at sufficiently late times brane-antibrane annihilation is no longer possible. The branes freeze out leaving a relic abundance of stable wrapped M2-branes. In [12] we studied the late-time evolution of such a universe both numerically and analytically and found that asymptotically the radii have simple power-law behavior. The late-time behavior depends on the number of wrapped branes, but is independent of the radii at freeze-out. Dimensions with no wrapped branes were indeed found to decompactify faster than all other dimensions.

Size of thermal fluctuations

In the brane gas scenario decompactification begins with a thermal fluctuation that causes $d$ of the spatial dimensions to expand. How likely is such
a fluctuation, as a function of $d$? To address this question we studied the semiclassical phase space for M-theory on $T^{10}$, and asked what fraction of the possible initial states lead to decompactification of $d$ dimensions [13]. Given the Hamiltonian constraint of general relativity, this is a problem in microcanonical statistical mechanics. We found that the expected number of decompactified dimensions depends strongly on the one parameter that cannot be fixed by thermodynamic reasoning, namely the volume of the initial-time hypersurface. For small initial volumes very few membranes are present in the initial state, and any that are present annihilate quickly. On the other hand for large initial volumes many membranes are present but their annihilation cross-section is small, and a non-zero fraction of the membranes typically freeze out as the universe expands. Only for a rather narrow range of initial volumes is there a significant probability of decompactifying three spatial dimensions.

**Holography and initial conditions**

To address the question of initial conditions we considered the restrictions imposed by the holographic principle, which states that the entropy in a given region is bounded by the surface area in Planck units. The semiclassical entropy density is fixed in terms of the Hubble parameter. The holographic principle therefore gives an upper bound on the size of the torus, as a function of the Hubble parameter. Somewhat discouragingly from the brane gas point of view, this bound on the volume makes it unlikely that three spatial dimensions will decompactify.

**String gas cosmology**

Easther, Green, Jackson and I went on to apply the techniques developed in [12, 13] to study the original string gas scenario of Brandenberger and Vafa. The results of [13] show that brane gas cosmology is unlikely to be a robust scenario for decompactifying three dimensions in M-theory. However there is reason to think that the scenario works better in weakly-coupled string theory. Essentially this is because dilaton gravity allows for a “loitering phase” in which the scale factors remain roughly constant, and this would seem to allow more time for a large thermal fluctuation to take place.

Our analysis shows that the typical outcome is in fact strongly dependent on the initial value of the string coupling. If one begins at fairly strong
coupling then very few strings are present in the initial state. Moreover the strings annihilate efficiently, and one typically ends up with a ten-dimensional radiation-dominated universe. On the other hand if one begins at weak coupling then large numbers of strings are present in the initial state, but the dilaton rolls to weak coupling so quickly that the strings freeze out before they have a chance to annihilate [14]. Thus many of the difficulties of brane gas cosmology are also present in weakly-coupled string theory.

**STRONGLY-COUPLED 0-BRANE QUANTUM MECHANICS**

Certain black holes in type IIA string theory can be described non-perturbatively in terms of the dynamics of $N$ D0-branes. In principle, this allows one to understand black hole physics in terms of a concrete many-body quantum mechanics problem. But in practice the quantum mechanics is strongly coupled and extremely difficult to work with.

Together with Gilad Lifschytz and David Lowe, I developed a mean-field approximation scheme for studying 0-brane dynamics [15]. This involved many technical challenges. To preserve supersymmetry, we were forced to develop a Hartree-Fock approximation in superspace. To incorporate the $U(N)$ gauge symmetry of the quantum mechanics, we worked in a physical supersymmetric gauge. The approximation gave results for thermodynamic quantities which are in quite good agreement with the black hole predictions, at least over a limited range of Hawking temperatures.

In collaboration with Nori Iizuka we went on to put a probe 0-brane in the black hole background [16]. Again using mean-field methods, we computed the effective potential for the probe, and found good agreement with supergravity predictions. We also computed the spectrum of single-string states, which showed that light states are present when the probe is close to the horizon of the black hole. This gave concrete support to a rather intriguing picture, in which absorption of a D-brane probe by the black hole is understood as thermal restoration of $U(N)$ gauge symmetry in the quantum mechanics [17].
In [16] Iizuka, Lifschytz, Lowe and I applied mean-field methods to study the strongly-coupled quantum mechanics of $N$ D0-branes at finite temperature. This quantum mechanics is dual to a ten dimensional black hole in a warped $AdS_2 \times S^8$ background. This work can be summarized by saying that the quantum mechanics has an effective description in terms of a non-interacting gas of quasiparticles. The quasiparticles are characterized by a mass and inverse lifetime of order the temperature.

In [18] we used this quasiparticle picture to make contact between the dual quantum mechanics and a stretched horizon [19] or membrane paradigm [20] description of the black hole. We identified the quasiparticles with the low-energy effective degrees of freedom of the membrane. Moreover the rate at which Hawking radiation is emitted from the stretched horizon is given by the Stefan-Boltzmann law, and is obviously proportional to the horizon area. This led to a simple explanation of the relation between the area of the horizon and the entropy of the dual quantum mechanics.

In [21] we generalized this quasiparticle picture to a wide variety of black hole and cosmological solutions. We showed that the same basic picture of the stretched horizon works for Schwarzschild black holes, black $p$-branes and rotating BTZ black holes, as well as for the cosmological horizon of de Sitter space. This shows that the quasiparticle description is valid in a much more general setting than the AdS/CFT context [18] where it was first derived.

References


