

Class format: we'll generally meet Mondays at 4pm in Pupin 908 and take turns presenting one of the following topics. Whoever does the presentation should assign a reasonable number of homework problems (say two or three), due back the following week. I'll post the assignments on www.phys.columbia.edu/~kabat/strings

The main reference is Zwiebach, *A first course in string theory*. I'd encourage you to glance through chapters 1 – 3 on your own. Sections 2.3 and 2.5 on light-front coordinates are particularly important, and the material on electrodynamics and gravity in general dimensions (sections 3.3 and 3.6) is useful background. I'd like to get through part I of Zwiebach this fall.

Zwiebach part I – the basics

1. Chapter 4: non-relativistic strings

The Lagrangian formulation is particularly important. Problems 4.3 and 4.6 look interesting to me.

2. Chapter 5: relativistic point particles

Important stuff. Problems 5.5 and 5.7 are classics.

3. Sections 6.1 – 6.5: relativistic strings – action and equations of motion

The basic framework of string theory.

4. Sections 6.6 – 6.9: more on relativistic strings

Further developments. Problems 6.4 and 6.7 look interesting.

5. Chapter 7: classical string dynamics

Develops some intuition for string dynamics. Lots of nice homework problems.

6. Chapter 8: worldsheet symmetry currents

Works out the various conserved quantities associated with string dynamics.

7. Chapter 9: classical strings in light-front gauge

The string equations of motion simplify in light-front gauge, something which helps a lot in the quantum theory.

8. Chapter 10: quantum fields in light-front gauge
A mini-course on quantum field theory in light-front gauge. I'd like to supplement this with a discussion of the Casimir effect.
9. Chapter 11: quantizing the relativistic particle
Quantizes the relativistic point particle in light-front gauge.
10. Sections 12.1 – 12.3: quantizing the relativistic open string
At last, quantum strings. They're just harmonic oscillators!
11. Sections 12.4 – 12.7: more on quantum open strings
Sections 12.4 and 12.6, discussing the Virasoro algebra and the string spectrum, are crucial. The mysterious (12.107) is just the Casimir energy of the worldsheet.
12. Chapter 13: quantum closed strings
Straightforward, given everything we've done for open strings. The new feature is that gravity appears in the string spectrum. We should probably leave the material on superstrings for later.

Zwiebach part II – further developments

1. D-branes and gauge fields: Zwiebach chapter 14. (important)
2. String charge and particle physics: Zwiebach chapter 15. (optional)
The material in sections 15.5 – 15.8, constructing the standard model on intersecting D6-branes, is new to me.
3. String thermodynamics and black holes: Zwiebach chapter 16. (optional)
4. T-duality of closed strings: Zwiebach chapter 17. (important)
5. T-duality of open strings and D-branes: Zwiebach chapter 18. (important)
6. Electromagnetic fields on D-branes: Zwiebach chapters 19 and 20. (optional)
7. Covariant string quantization: Zwiebach chapter 21. (optional)

Beyond Zwiebach

I'd also like to discuss vertex operators, scattering amplitudes, and a bit of superstring theory. Some of this is in Zwiebach – the Veneziano amplitude appears in section 22.8, and superstrings are mentioned in section 13.5 – but for these topics volume I of Green, Schwarz, Witten is a better reference. Here's an outline of topics we could try to cover. *Some exposure to path integrals and complex analysis may be necessary.*

1. The Polyakov action

This is discussed in Zwiebach section 21.6, but his discussion seems a bit backwards to me. I'd be tempted to start by postulating (21.70) and then show that it's equivalent to the Nambu-Goto action.

2. Wick rotation

In the Polyakov approach it would seem one should take the auxiliary worldsheet metric $h_{\alpha\beta}$ to be Lorentzian. However one typically Wick rotates and turns it into a Euclidean metric. This has always seemed a bit fishy to me.¹ There's some discussion of this point in Polchinski vol. I pp. 82 – 83. It's also mentioned in passing by GSW on p. 124. In any case it leads to the representation of string worldsheets as Riemann surfaces discussed in Zwiebach chapter 22.

3. Strings in background fields

This is discussed in GSW section 3.4.5. It leads to a nice understanding of the relation between the dilaton expectation value and the string coupling constant, see GSW section 3.4.6.

4. Vertex operators

This is mentioned in GSW section 1.4.2, although they just guess the operators. Polchinski pp. 63 – 64 is more explicit, although I find his discussion a bit obscure. I wrote up some notes on this once which may be useful.

5. Tree-level scattering

GSW section 1.4.3 presents the gauge-fixed path integral. Scattering of closed string tachyons (the Virasoro amplitude) is treated in section 1.4.4, and scattering of open-string tachyons (the Veneziano amplitude) is covered on pp. 47–50. A few comments on the treatment:

¹My guess is that to make it precise one would have to use the light-cone formalism of GSW chapter 11.

- In some ways open string tachyons are simpler, it would be nice if GSW did them first.
- In the intro GSW is sketchy about the gauge-fixing procedure for the global conformal group. You can find the details on p. 369 for open strings ($SL(2, \mathbb{R})$) and pp. 384 – 385 for closed strings ($SL(2, \mathbb{C})$).

Zwiebach just postulates the Veneziano amplitude on p. 512. It has many remarkable features which were really the starting point for string theory.

- pole structure (Zwiebach p. 515)
- channel duality (GSW section 1.1)
- Regge behavior in the large s , fixed t limit (GSW p. 11)
- fixed angle scattering is exponentially suppressed (GSW p. 12): string scattering is "softer" than any scattering process in quantum field theory!

6. Loop amplitudes (optional material)

The discussion of the moduli space of tori in Zwiebach section 23.6 is very relevant. The final result is in Polchinski, equation (7.3.6) where \mathcal{F}_0 is the fundamental domain described in Zwiebach and the Dedekind eta function is defined in Polchinski equation (7.2.43). The remarkable thing is that string loops are free from the ultraviolet divergences which plague ordinary quantum field theories.

7. Superstrings I

GSW section 4.1. The material on fermions and supersymmetry in 1+1 dimensions is important background.

8. Superstrings II

GSW sections 4.2 and 4.3. We should get through the GSO conditions in section 4.3.3, but that's probably a good stopping point.