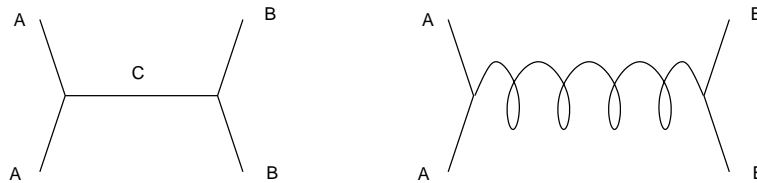


1. Unitarity bounds

Recall the scalar-exchange and graviton-exchange scattering amplitudes for $AA \rightarrow BB$ scattering from the last homework.



- (i) Compute the partial wave amplitudes \mathcal{M}_J and partial wave cross sections σ_J associated with the scalar exchange amplitude $\mathcal{M}_{\text{scalar}}$. For what J are they non-zero?
- (ii) Show that the high-energy behavior of the scalar-exchange amplitude is compatible with unitarity. (You can understand this as related to Kurt's comment, that the couplings g_{AAC} and g_{BBC} have units of mass.)
- (iii) Compute the partial wave amplitudes \mathcal{M}_J and partial wave cross sections σ_J associated with the graviton exchange amplitude $\mathcal{M}_{\text{grav}}$. For what J are they non-zero?
- (iv) Show that the graviton-exchange amplitude violates unitarity at high energy. At what center-of-mass energy does unitarity first break down? Give your answer in GeV.

2. Green's function

Show that $G(x) = \frac{1}{2\pi} \log(\mu|x|)$ is a Green's function for the Laplace operator in a flat, two-dimensional Euclidean space. That is, show that $\nabla^2 G(x) = \delta^2(x)$. Here μ is an arbitrary parameter with units of 1/length that makes the argument of the log dimensionless.

(This problem can be solved in many ways. Perhaps the simplest is to construct the Coulomb potential associated with a point charge in 2+1 dimensions.)