The MSSM

\[ W_{\text{MSSM}} = \mu H_u H_d + \lambda_{ij}^u Q_i \bar{u}_j H_u + \lambda_{ij}^d Q_i \bar{d}_j H_d + \lambda_{ie}^L \bar{Q}_i e_j H_d \]

I’ve suppressed all gauge indices. There’s a unique way of contracting them (with two-index \( \epsilon \)-tensors if need be) to make the superpotential gauge invariant.

---

**Table:**

<table>
<thead>
<tr>
<th>superfield</th>
<th>( SU(3)_C \times SU(2)_L \times U(1)_Y )</th>
<th>particle content</th>
<th>R-parity</th>
</tr>
</thead>
<tbody>
<tr>
<td>vector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_3 )</td>
<td>((8, 1, 0))</td>
<td>gluons ( g ) and gluinos ( \tilde{g} )</td>
<td>+</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>((1, 3, 0))</td>
<td>W-bosons ( W ) and winos ( \tilde{W} )</td>
<td>+</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>((1, 1, 0))</td>
<td>hypercharge boson ( B ) and bino ( \tilde{B} )</td>
<td>+</td>
</tr>
<tr>
<td>chiral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L_i )</td>
<td>((1, 2, -1))</td>
<td>left-handed leptons ( \left( \nu_e \right)_{Li} ) and sleptons ( \left( \tilde{\nu}<em>e \right)</em>{Li} )</td>
<td>-</td>
</tr>
<tr>
<td>( \tilde{e}_i )</td>
<td>((1, 1, 2))</td>
<td>right-handed leptons ( \tilde{e}<em>{Ri} ) and sleptons ( \tilde{\nu}</em>{Ri} )</td>
<td>-</td>
</tr>
<tr>
<td>( Q_i )</td>
<td>((3, 2, 1/3))</td>
<td>left-handed quarks ( \left( u \right)<em>{Li} ) and squarks ( \left( \tilde{u} \right)</em>{Li} )</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{u}_i )</td>
<td>((3, 1, -4/3))</td>
<td>right-handed quarks ( \bar{u}<em>{Ri} ) and squarks ( \bar{\nu}</em>{Ri} )</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{d}_i )</td>
<td>((3, 1, 2/3))</td>
<td>right-handed quarks ( \bar{d}<em>{Ri} ) and squarks ( \bar{\nu}</em>{Ri} )</td>
<td>-</td>
</tr>
<tr>
<td>( H_u )</td>
<td>((1, 2, 1))</td>
<td>up-type Higgs ( \left( H^+_u \right) ) and Higgsinos ( \left( H^0_u \right) )</td>
<td>+</td>
</tr>
<tr>
<td>( H_d )</td>
<td>((1, 2, -1))</td>
<td>down-type Higgs ( \left( H^0_d \right) ) and Higgsinos ( \left( H^+_d \right) )</td>
<td>+</td>
</tr>
</tbody>
</table>

Notation: \( i = 1, 2, 3 \) is a generation index. The superfields \( \tilde{e}_i, \bar{u}_i, \bar{d}_i \) are chiral (not anti-chiral) – the overbar is part of their name, it doesn’t indicate complex conjugation. The component fields \( \tilde{e}_R, \bar{u}_R, \bar{d}_R \) are left-handed chiral spinors – the overbar indicates that they’re related by complex conjugation to the right-handed components of a Dirac spinor. The squarks and sleptons are all complex scalar fields; rather than indicating helicity the subscripts \( L, R \) tell which fermion they’re related to by supersymmetry.
Soft terms

We add the following soft susy-breaking terms to the MSSM Lagrangian.

\[-L_{\text{soft}} = (m_{\tilde{q}}^2)^{ij} \tilde{q}_i \tilde{q}_j + (m_{\tilde{u}}^2)^{ij} \tilde{u}_i \tilde{u}_j + (m_{\tilde{d}}^2)^{ij} \tilde{d}_i \tilde{d}_j + (m_{\tilde{e}}^2)^{ij} \tilde{e}_i \tilde{e}_j + m_{\tilde{h}}^2 \tilde{h}_i \tilde{h}_j + (B \mu) \tilde{h}_i \tilde{h}_j + \text{c.c.} + (2m_3 \text{Tr}(\tilde{g}\tilde{g}) + 2m_2 \text{Tr}(\tilde{W}\tilde{W}) + m_1 \tilde{B}\tilde{B} + \text{c.c.}) + (A_{\tilde{q}}^{ij} \tilde{q}_i \tilde{u}_j^* + A_{\tilde{d}}^{ij} \tilde{d}_i \tilde{d}_j^* + A_{\tilde{e}}^{ij} \tilde{e}_i \tilde{e}_j^* + \text{c.c.}) + (C_{\tilde{q}}^{ij} \tilde{q}_i \tilde{d}_j^* + C_{\tilde{d}}^{ij} \tilde{d}_i \tilde{u}_j^* + C_{\tilde{e}}^{ij} \tilde{e}_i \tilde{e}_j^* + \text{c.c.})\]

Notation: \( \tilde{q}_i = (\tilde{u}_i \tilde{d}_i)_L \) are the left-handed squarks, \( \tilde{u}_i = \tilde{u}_{R_i} \) and \( \tilde{d}_i = \tilde{d}_{R_i} \) are the right-handed squarks, \( \tilde{l}_i = (\tilde{\nu} \tilde{e})_L \) are the left-handed sleptons, \( \tilde{e}_i = \tilde{e}_{R_i} \) are the right-handed sleptons, \( h_u = (H_u^+ H_u^-)_L \) and \( h_d = (H_d^+ H_d^-)_L \) are the Higgs doublets. Remember it’s the complex conjugates of \( \tilde{u}_i, \tilde{d}_i, \tilde{e}_i \) that sit in chiral superfields. For the most part people neglect the \( C \) terms.

Higgs spectrum

After electroweak symmetry breaking we have

\[
\langle h_u \rangle = \begin{pmatrix} 0 \\ v_u/\sqrt{2} \end{pmatrix}, \quad \langle h_d \rangle = \begin{pmatrix} v_d/\sqrt{2} \\ 0 \end{pmatrix}
\]

The ratio of Higgs vevs is denoted \( \tan \beta = v_u/v_d \), while \( v = \sqrt{v_u^2 + v_d^2} \) replaces the standard model Higgs vev in gauge boson masses. To expand about this vacuum we set

\[
h_u = \begin{pmatrix} H^+ \cos \beta \\ \frac{1}{\sqrt{2}}(v_u + \tilde{H}_u + iA^0 \cos \beta) \end{pmatrix}, \quad h_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \tilde{H}_d + iA^0 \sin \beta) \\ H^- \sin \beta \end{pmatrix}
\]

where \( \tilde{H}_u, \tilde{H}_d, A^0 \) are real and \( H^+ = (H^-)^* \) is complex.

The neutral real Higgs scalar \( A^0 \) acquires a mass

\[
m_{A^0}^2 = 2B\mu/\sin 2\beta.
\]
The fields $\hat{H}_u, \hat{H}_d$ mix to form light and heavy neutral Higgs scalars $h^0, H^0$.

$$m_{h^0,H^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_{A^0}^2m_Z^2 \cos^2 2\beta} \right)$$

Finally the charged Higgs fields have a mass

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2.$$

**Standard model particle spectrum**

The $W$ and $Z$ bosons couple to both Higgs doublets, so their masses are

$$m_W^2 = \frac{1}{4} g_2^2 (v_u^2 + v_d^2) = \frac{1}{4} g_2^2 v^2$$

$$m_Z^2 = \frac{1}{4} \left( g_1^2 + g_2^2 \right) \left( v_u^2 + v_d^2 \right) = \frac{1}{4} g_Z^2 v^2$$

The up-type quarks get a mass from the $H_u$ Higgs doublet, for example the top mass is

$$m_t = \frac{1}{\sqrt{2}} \lambda_t v_u.$$  

The charged leptons and down-type quarks get a mass from the $H_d$ Higgs doublet, for example the bottom mass is

$$m_b = \frac{1}{\sqrt{2}} \lambda_b v_d.$$  

**Neutralinos**

The four neutral fermions $N = \left( \tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0 \right)^T$ mix to form neutralinos $\tilde{\chi}^0_i, i = 1, 2, 3, 4$. The mass matrix is

$$\mathcal{L} = -\frac{1}{2} N^T M N + \text{c.c.}$$

$$M = \begin{pmatrix} m_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & m_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}$$

Here $c_\beta = \cos \beta$, $c_W = \cos \theta_W$, etc.
Charginos

The charged fermions $C^+ = \left( \tilde{W}^+_+ \tilde{H}_u^+ \right)$, $C^- = \left( \tilde{W}^-_- \tilde{H}_d^- \right)$ mix to form charginos $\tilde{\chi}^\pm_i$, $i = 1, 2$. The mass matrix is (note that $C^+ \neq (C^-)^*$)

$$\mathcal{L} = -C^{-T} X C^+ + \text{c.c.}$$

$$X = \begin{pmatrix} m_2 & \sqrt{2}m_W s_\beta \\ \sqrt{2}m_W c_\beta & \mu \end{pmatrix}$$

You can diagonalize $X = U^T \left( \begin{array}{cc} m_{\tilde{\chi}_1} & 0 \\ 0 & m_{\tilde{\chi}_2} \end{array} \right) V$ using $2 \times 2$ unitary matrices $U, V$.

Gluinos

The gluinos can’t mix with any other MSSM particles. They get a mass from the soft terms, $\mathcal{L} = -2m_3 \text{Tr}(\tilde{g}\tilde{g}) + \text{c.c.}$

Squarks

Neglecting inter-generational mixing, the two top squarks $\tilde{t}_{L,R}$ will mix to form mass eigenstates $\tilde{t}_{1,2}$. Their mass matrix is

$$\mathcal{L} = - \left( \begin{array}{cc} \tilde{t}_L^* & \tilde{t}_R^* \end{array} \right) M^2 \left( \begin{array}{c} \tilde{t}_L \\ \tilde{t}_R \end{array} \right)$$

$$M^2 = \begin{pmatrix} m_{\tilde{q}_t}^2 + m_{\tilde{t}}^2 + \Delta_t & v_u A_t - \mu v_d \lambda_t \\ v_u A_t - \mu v_d \lambda_t & m_{\tilde{t}}^2 + m_{\tilde{b}}^2 + \Delta_\tilde{b} \end{pmatrix}$$

where the $D$-term contributions are $\Delta_t,i = (T^3_L - Q_{e.m.} \sin^2 \theta_W) m_Z^2 \cos 2\beta$. The mixing often makes $\tilde{t}_1$ the lightest squark.

For the remaining squarks the mixing is smaller. For the most part their masses arise directly from the mass terms in $L_{\text{soft}}$. 
Neglecting inter-generational mixing, the two tau sleptons $\tilde{\tau}_{L,R}$ will mix to form mass eigenstates $\tilde{\tau}_{1,2}$. Their mass matrix is

\[
\mathcal{L} = - \begin{pmatrix} \tilde{\tau}_L^* & \tilde{\tau}_R^* \end{pmatrix} M^2 \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}
\]

\[
M^2 = \begin{pmatrix} m_{\tilde{\tau}}^2 + m_{\tilde{\tau}}^2 + \Delta_{\tilde{\tau}} & v_d A_{\tilde{\tau}} - \mu v_u \lambda_{\tilde{\tau}} \\ v_d A_{\tilde{\tau}} - \mu v_u \lambda_{\tilde{\tau}} & m_{\tilde{\tau}}^2 + m_{\tilde{\bar{\tau}}}^2 + \Delta_{\tilde{\bar{\tau}}} \end{pmatrix}
\]

where again the $D$-term contributions are $\Delta_{\tau,\bar{\tau}} = (T^3_L - Q_{e.m.} \sin^2 \theta_W) m_Z^2 \cos 2\beta$. Due to mixing $\tilde{\tau}_1$ is often the lightest slepton.

For the remaining sleptons the mixing is smaller. For the most part their masses arise directly from the mass terms in $\mathcal{L}_{\text{soft}}$. 

Sleptons