Two pion scattering, $I=0$ and disconnected diagrams in Lattice QCD

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1 Introduction

2 $\eta$, & $\eta'$ masses
   - Diagonalize the $|l\rangle$ and $|s\rangle$ states method
   - Octet and singlet operator Method

3 $\sigma$ mass

4 $\pi\pi$ scattering

5 conclusion
Disconnected diagrams

- $\epsilon'$ from $K$ to $\pi\pi$ $\Delta I=1/2$

- $\pi\pi$ scattering in Isospin 0 channel

- Two-quark states $\eta$ & $\eta'$, $\sigma$
Introduction

Computation specifics

- **Lattice:**
  - 2+1 flavor DWF, $m_s = 0.04$, $m_l = 0.01$, (also 0.02, 0.03)
  - Iwasaki gauge action $\beta = 2.13$
  - $a^{-1} = 1.73(3)\,\text{GeV}$.
  - $16^3 \times 32$ space time volume with $L_s = 16$

- **Propagators:** $D_w(t_{\text{sink}}; t_{\text{src}})$
  - Coulomb Gauge Fixed Wall Source and Sink
  - Propagators are calculated on All time slices $T=32$ ($\times 12$ inversion)
    → Huge statistics $\times 150$ configurations
    → Resolve signal from disconnect diagram
  - Eigenvector accelerator code, provided by Ran Zhou (6 hours $\rightarrow$ 3.7 hours)
  - Correlation function $\leftarrow$ Take the contraction
With the approximate SU(3) flavor symmetry,

\[
|\eta\rangle \approx \frac{1}{\sqrt{6}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s) : \text{Octet} \quad (1)
\]

\[
|\eta'\rangle \approx \frac{1}{\sqrt{3}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s) : \text{Singlet} \quad (2)
\]
Diagonalize the $|l\rangle$ and $|s\rangle$ states method

**Definition**

\[ |l\rangle = \frac{1}{\sqrt{2}} (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) \]  
\[ |s\rangle = \bar{s}\gamma_5 s \]  

Diagonalize the correlation matrix: [direct VS. Lüscher Wolff Method]

\[
\begin{pmatrix}
\langle l(t)l^\dagger(0) \rangle & \langle s(t)l^\dagger(0) \rangle \\
\langle l(t)s^\dagger(0) \rangle & \langle s(t)s^\dagger(0) \rangle \\
\end{pmatrix}
= \begin{pmatrix}
C_l - 2D_l & -\sqrt{2}D_x \\
-\sqrt{2}D_x & C_s - D_s \\
\end{pmatrix}
\]

We could expect to get $\eta$ & $\eta'$ states.

\[ |\eta\rangle = \cos \theta |l\rangle - \sin \theta |s\rangle \]  
\[ |\eta'\rangle = \sin \theta |l\rangle + \cos \theta |s\rangle \]  

SU(3) flavor symmetry limit: \( \sin \theta = \sqrt{\frac{2}{3}} = 0.8165, \cos \theta = \sqrt{\frac{1}{3}} \),
Diagonalize the $|l\rangle$ and $|s\rangle$ states method

All 5 contractions

$C_l$: 

$C_s$: 

$D_l$: 

$D_s$: 

$D_x$: 

Qi Liu (Columbia University, RBC and UKQCD Collaborations)
$$m_\eta = 0.401(11) = 694(19)\text{MeV}$$

$$m_{\eta'} = 0.653(82) = 1.13(14)\text{GeV}$$

Diagonalize the $|l\rangle$ and $|s\rangle$ states method.
\[ \eta \sim \eta' \] masses

Diagonalize the \( |l\rangle \) and \( |s\rangle \) states method

**Mixing angle**

\[ |\eta\rangle = \cos \theta |l\rangle - \sin \theta |s\rangle \]

\[ |\eta'\rangle = \sin \theta |l\rangle + \cos \theta |s\rangle \]

SU(3) flavor symmetry limit:

\[ \sin \theta = \sqrt{\frac{2}{3}} = 0.8165 \]

\[ \cos \theta = \sqrt{\frac{1}{3}} = 0.5774 \]
$\eta, \eta'$ correlation function

$$|\eta\rangle = \frac{1}{\sqrt{6}} (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d - 2\bar{s} \gamma_5 s) : \text{Octet} \quad (7)$$

$$|\eta'\rangle = \frac{1}{\sqrt{3}} (\bar{u} \gamma_5 u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s) : \text{singlet} \quad (8)$$

Expand the correlation function in terms of possible diagrams, we have:

$$\langle \eta(t)\eta^\dagger(0) \rangle = \frac{1}{3} (C_l + 2C_s) - \frac{2}{3} (D_l + D_s - 2D_x) \quad (9)$$

$$\langle \eta'(t)\eta'^\dagger(0) \rangle = \frac{1}{3} (2C_l + C_s) - \frac{1}{3} (4D_l + D_s + 4D_x) \quad (10)$$

$$m_\eta = 0.392(12) = 678(20) \text{MeV}, \quad m_{\eta'} = 0.671(82) = 1.16(14) \text{GeV}$$

They are almost Orthogonal.
Compare with chiral perturbation theory

\[ m_\pi = 0.2472(10) = 427.7 \text{MeV}, \ m_k = 0.3560(20) = 615.9 \text{MeV} \]

From the chiral perturbation theory, we have

\[ 3m_\eta^2 + m_\pi^2 = 4m_k^2 \]

We can calculate the theoretical value for the \( \eta \) mass

\[ m_\eta^0 = 0.3855(25) = 667(4) \text{MeV} \]

Compare with our result:

1. Diagonalize \(|l\rangle, |s\rangle\): \( m_\eta = 0.401(11) = 694(19) \text{MeV} \)
2. Octet operator: \( m_\eta = 0.392(12) = 678(20) \text{MeV} \)
Extrapolate to physical pion mass

Notice: Strange quark mass is fixed. (Extrapolate to a Kaon mass \( \approx 550\text{MeV} \))
\[ \sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d), \text{ it may also mixes with } \bar{s}s \text{ state}. \]

Mixing angle \( \sin(\theta) \approx 0.13 \)
$m_\sigma = 0.49(11) [4 : 9]$
Extrapolate to physical pion mass

Notice: Strange quark mass is fixed.
\[ t < 20|20 >_0 = 2(D - C) \quad (11) \]
\[ t < 00|00 >_0 = 2D + C - 6R + 3V \quad (12) \]
Use Lüscher formula to calculate the scattering length:

$$\Delta E = E - 2m_\pi = -\frac{4\pi a_0}{m_\pi L^3} [1 + c_1 \frac{a_0}{L} + c_2 (\frac{a_0}{L})^2] + O(1/L^6)$$

<table>
<thead>
<tr>
<th>$m_1(\text{conf})$</th>
<th>$m_\pi$</th>
<th>$\Delta E(I = 2)$</th>
<th>$\Delta E(I = 0)$</th>
<th>$\Delta E(I = 0 \text{Vout})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01(150)</td>
<td>0.2472(10)</td>
<td>0.0197(14)</td>
<td>-0.041(80)</td>
<td>-0.0439(72)</td>
</tr>
<tr>
<td>0.02(150)</td>
<td>0.3248(9)</td>
<td>0.0158(7)</td>
<td>-0.082(64)</td>
<td>-0.0268(69)</td>
</tr>
<tr>
<td>0.03(281)</td>
<td>0.3895(6)</td>
<td>0.0135(5)</td>
<td>0.04(10)</td>
<td>-0.0155(50)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$m_\pi$</th>
<th>$a_0(I = 2)$</th>
<th>$a_0(I = 0)$</th>
<th>$a_0(I = 0 \text{Vout})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2472(10)</td>
<td>-1.26(7)</td>
<td>4.6 [-2.1:8.1]</td>
<td>4.9 [4.2:5.4]</td>
</tr>
<tr>
<td>0.3248(9)</td>
<td>-1.31(5)</td>
<td>7.7 [2.7:9.6]</td>
<td>4.1 [3.0:4.9]</td>
</tr>
<tr>
<td>0.3895(6)</td>
<td>-1.34(5)</td>
<td>-2.9 [-6.0:7.3]</td>
<td>2.8 [1.7:3.8]</td>
</tr>
</tbody>
</table>
Compare with other results

Tree level $\chi$PT (Current Algebra Formula):

$$a_0(I = 2) = -\frac{1}{16\pi} \frac{2m_\pi^2}{f_\pi^2}, a_0(I = 0) = \frac{1}{16\pi} \frac{7m_\pi^2}{f_\pi^2}$$

$$f_\pi = 0.0765 + 1.02(m_l + 0.00308), \quad m_\pi^2 = 2 \times 2.285(m_l + 0.00308)$$

from hep-ph/0612112V1
Conclusion

What did we get?

- Good result for $\eta$,
- Big error for $\eta'$ and $\sigma$
- Huge error for $I=0$ $\pi\pi$ scattering. (Need the energy difference).

Are we able to calculate $\epsilon'$ from $k$ to $\pi\pi$?

- Large volume
  $\rightarrow$ reduced vacuum noise
- Physical $\pi$ mass(smaller)
  $\rightarrow$ Signal decays slower
  $\rightarrow$ Large time separation.
Thank you!
\[ m_\eta = 0.392(12) = 678(20)\text{MeV} \]

\[ m_{\eta'} = 0.671(82) = 1.16(14)\text{GeV} \]

fitting range \([5:12]\) octet

fitting range \([3:7]\) singlet
Octet state and singlet state mixing angle to get $\eta$ and $\eta'$

$\text{Value} = \sin(\theta) \cos(\theta)$

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