Experimental errors

1.1 Why do we do experiments?
There are basically two different types of results of experiments that scientists perform in order to learn about the physical world. In one type, we set out to determine the numerical value of some physical quantity, while in the second we are testing whether a particular theory is consistent with our data. These two types are referred to as 'parameter determination' and 'hypothesis testing' respectively. (Of course, in real life situations there is a degree of overlap between the two: a parameter determination may well involve the assumption that a specific theory is correct, while a particular theory may predict the value of a parameter.) For example, a parameter determination experiment could consist of measuring the velocity of light, while a hypothesis testing experiment could check whether the velocity of light has suddenly increased by several percent since the beginning of this year.

In this chapter, we are mainly concerned with various aspects of calculating the accuracy of parameter determination type experiments. We will have more to say about hypothesis testing experiments in Chapter 2.

1.2 Why estimate errors?
When we performed parameter determination experiments at school, we considered that the job was over once we obtained a numerical value for the quantity we were trying to measure. At university, and even more so in every-day situations in the laboratory, we are concerned not only with the answer but also with its accuracy. This accuracy is expressed by quoting an experimental error on the quantity of interest. Thus a determination of the velocity of light might yield an answer

\[ c = (3.09 \pm 0.15) \times 10^8 \text{ metres/sec.} \]

In Section 1.4, we will say more specifically what we mean by the error
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Thus for a given value of the parameter, our reaction - 'Conventional physics is in good shape' or 'We have made a world shattering discovery' or 'We should find out how to do better experiments' - in this case depends on the numerical estimate of the accuracy of our experiment. Conversely, if we know only that the result of the experiment is that the value of $c$ was determined as $3.09 \times 10^8$ metres/sec (but do not know the value of the experimental error), then we are completely unable to judge the significance of this result.

The moral is clear. Whenever you determine a parameter, estimate the error or your experiment is useless.

1.3 Random and systematic errors

There are two fundamentally different sorts of errors associated with any measurement procedure, namely random and systematic errors. The random errors come simply from the inability of any measuring device to give infinitely accurate answers, while the systematic errors are more in the nature of mistakes.

These different types of error are illustrated by considering an experiment involving counters to determine the decay constant $\lambda$ of a radioactive source. The necessary measurements consist of counting how many decays are observed in a given time in order to determine the decay rate $(-dn/dt)$, and of weighing the sample (in order to obtain the number of nuclei $n$ present). Then the decay constant $\lambda$ is calculated from the formula

$$\frac{dn}{dt} = \lambda n.$$ (1.1)

The main random error comes from the fact that there is an inherent statistical error involved in counting random events. Other contributions to the random error come from the timing of the period for which the decays are observed and from the uncertainty in the mass of the sample.

The more obvious possible sources of systematic error are:

(i) The counters used for detecting the decays may not be completely efficient and/or they may not completely surround the source, and so the counting rate may be below the true decay rate.

(ii) The counters may be sensitive to particles coming from other than the source (e.g. cosmic rays), which would give a counting rate above the true decay rate.

(iii) The radioactive source may not be pure (either chemically or

† Since this is an experimental number, it too has an experimental uncertainty, but $c$ has been measured so well that the uncertainty arises in the eighth figure after the decimal point, so we can effectively forget about it here.
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all the apparatus to be at a constant temperature; or we really ought to be performing our experiment in a vacuum instead of in the laboratory atmosphere; or the presence of some impurity is producing spurious results. Ideally, of course, all such effects should be absent. But if it is known that such a distorting effect is present, than at least some attempt can be made to estimate its importance and to correct for it. In effect, we are then converting what was previously a systematic error into what is hopefully only a random one.

In general, there are no simple rules or prescriptions for eliminating systematic errors. To a large extent it is simply a matter of common sense plus experience to know what are the possible dangerous sources of errors of this type. But one possible check that can sometimes be helpful is to use constraints that may be relevant to the particular problem. We give three simple illustrations of this.

**Example (a)**

We want to know whether a certain protractor has been correctly calibrated. One possible test is to use this protractor to measure the sum of the angles of a triangle. We then repeat this for several triangles and plot a histogram of our results. If the resulting distribution peaks significantly away from 180°, our protractor may be in error.

**Example (b)**

If a radioactive source emits electrons and photons in coincidence, then (after we have corrected for the inefficiencies of the various counters) we should observe the same number of counts in our electron as in our photon detector. A similar check could be made for a particle whose branching ratios for decays to electrons and to muons are equal.

A recurring theme in this book is the necessity of providing error estimates on any measured quantity. Because of their nature, random errors will make themselves apparent by producing somewhat different values of the measured parameter in a series of repeated measurements. The estimated accuracy of the parameter can then be obtained from the spread in measured values as described in Section 1.4.2.

An alternative method of estimating the accuracy of the answer exists in cases where the spread of measurements arises because of the limited accuracy of measuring devices. The estimates of the uncertainties of such individual measurements can be combined as explained in Section 1.5 in order to derive the uncertainty of the final calculated parameter. This approach can be used in situations where a repeated set of measurements is not available to employ the method described in the previous paragraph. In cases where both approaches can be used, they should of course yield consistent answers.

The accuracy of our measurements will of course play little part in determining the accuracy of the final parameter in those situations in which the population on which the measurements are made exhibits its own natural spread of values. For example, the heights of ten-year-old children will in general be scattered by an amount which is larger than the uncertainty with which the height of any single child can be measured.

An intermediate situation arises where the observation consists in counting independent random events in a given interval. Although the spread of values will usually be larger than the accuracy of counting (which may well be exact), it is known (see Section 3.2) that for an expected number of observations \( n \), the spread is \( \sqrt{n} \).

For systematic errors, however, the ‘repeated measurement’ approach will not work; if our detector, which we believe counts all particles passing through it, in fact has an efficiency of only 1%, the decay rate will come out too small by a factor of \( \sim 100 \) each time we repeat the experiment, and yet everything will look consistent.

Systematic errors can arise on any of the actual measurements that are required in order to calculate the final answer. But they can also appear indirectly in variables which do not explicitly appear in any formulae. Thus, for example, maybe the derivation of the formula being used requires isotopically) and so the number of nuclei capable of decaying may be less than the number as deduced from the mass of the sample.

(iv) There may also be calibration errors in the clock and balance used for measuring the time interval and the sample mass.