OPTICAL PUMPING

I. Preliminary

This experiment involves techniques and phenomena many of which were initially explored by members of the Columbia Physics department. Prof. I. I. Rabi and his colleagues, used R. F. spin flip techniques in atomic beam experiments. The “Breit-Rabi” formula used is from Phys. Rev. 38, 2082 (1931). The $|g|$ factor for the electron spin is 2.0 vs. 1.0 for the orbital momentum in Dirac’s theory of the electron. The Kush-Foley studies (Phys. Rev. 72, 1256 (1947)) first established the anomalous value $g_s \approx 2.0023$. This, along with Columbia results of W. E. Lamb and Rutherford on the hydrogen fine structure, provided the theoretical stimulus for the development of Quantum Electrodynamics as it is today. Professors R. Novick, S. Hartmann, and W. Happer were active in developing the field of optical pumping.

II. Theory of the Atomic States in an External Magnetic Field

This experiment studies the optical pumping of Rubidium between the ground state $5^2S_{1/2}$ level and the $5^2P_{1/2}$ level. Natural Rb is 72.15% $^{85}$Rb, which has $I = 5/2, \mu = +1.352 \mu_N$ and 27.85% $^{87}$Rb, which has $I = 3/2, \mu = +2.750 \mu_N$. Here $I$ is the nuclear angular momentum in units of $\hbar$, and $\mu_N$ is the nuclear magneton, which is smaller than the Bohr magneton $\mu_B$ by a factor 1836.1. The magneton sizes are, for this experiment, best expressed in units of precession frequency (Hz/Gauss):

$$\mu_N = 762.270 \text{ Hz/Gauss, } \mu_B = 1.399611 \cdot 10^6 \text{ Hz/Gauss}$$

$$\mu_B \equiv e\hbar / 2m_e c, \ \mu_N \equiv e\hbar / 2m_p c.$$  

The ground states of the alkali metal atoms are closed shells plus one valence electron in the next $s$ state ($\ell = 0$). These are $2S, 3S, 4S, 5S, 6S$ for Li, Na, K, Rb, and Cs, respectively. We are interested in the transitions between the ground state and the excited states having the electron in the $P$ state (same $n$, $\ell = 1$). For Na, the transitions $3P_{3/2} \rightarrow 3S_{1/2}$ and $3P_{1/2} \rightarrow 3S_{1/2}$ are the strong sodium yellow lines at 5890 and 5896 Å, respectively, called the “D” lines. The $P_{3/2} \rightarrow S_{1/2}$ is called the D$_2$ and the $P_{1/2} \rightarrow S_{1/2}$ is called the D$_1$ line. Their energy separation is due to the spin orbit coupling $\vec{\ell} \cdot \vec{s}$. The same notation is used for the other alkali metals. The D lines fractional fine structure splitting $\Delta \lambda / \lambda_{\text{mean}}$ increases rapidly with the atomic number $Z$. For Li it is about 1/45000, for Na about 1/1000, for K about 1/226, for Rb about 1/53 and for Cs about 1/21. In Table 1, $\lambda_1$ and $\lambda_2$ are the D$_1$ and D$_2$ line wavelengths.
### Table 1 - Basic atomic parameters of alkali atoms

<table>
<thead>
<tr>
<th>Atom</th>
<th>abundance %</th>
<th>( I )</th>
<th>( \Delta \nu ) (MHz)</th>
<th>( \lambda_1 ) (Å)</th>
<th>( \lambda_2 ) (Å)</th>
<th>( \mu_I ) (Nuclear magnetons)</th>
<th>( g_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{6} \text{Li} )</td>
<td>7.5</td>
<td>1</td>
<td>228.2</td>
<td>6708</td>
<td>6708</td>
<td>0.8220</td>
<td></td>
</tr>
<tr>
<td>( ^{7} \text{Li} )</td>
<td>92.5</td>
<td>3/2</td>
<td>803.5</td>
<td>3 τ</td>
<td>2564</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ^{22} \text{Na} )</td>
<td>100</td>
<td>3/2</td>
<td>1771.6</td>
<td>5896</td>
<td>5890</td>
<td>2.2176</td>
<td></td>
</tr>
<tr>
<td>( ^{39} \text{K} )</td>
<td>93.2</td>
<td>3/2</td>
<td>461.7</td>
<td>7699</td>
<td>7665</td>
<td>0.3914</td>
<td></td>
</tr>
<tr>
<td>( ^{41} \text{K} )</td>
<td>6.8</td>
<td>3/2</td>
<td>254.0</td>
<td>0.8</td>
<td>2148</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ^{85} \text{Rb} )</td>
<td>72.2</td>
<td>5/2</td>
<td>3035.7</td>
<td>7948</td>
<td>7800</td>
<td>1.3527</td>
<td>0.000295</td>
</tr>
<tr>
<td>( ^{87} \text{Rb} )</td>
<td>27.8</td>
<td>3/2</td>
<td>6834.7</td>
<td>2.7506</td>
<td>0.000999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ^{183} \text{Cs} )</td>
<td>100</td>
<td>7/2</td>
<td>9192.6</td>
<td>8944</td>
<td>8521</td>
<td>2.579</td>
<td></td>
</tr>
</tbody>
</table>

The electron angular momentum \( \vec{J} = \vec{\ell} + \vec{s} \) couples with the nuclear angular momentum to form a total angular momentum \( \vec{F} = \vec{J} + \vec{I} \). For the \( ^2S_{1/2} \) ground state, this leads to \( F = (I + 1/2) \) and \( F = (I - 1/2) \) with the \( (I - 1/2) \) state lower as shown in Fig. 1. The hyperfine separation of these two states is given by \( \Delta \nu \) in Table 1. For \( ^{85}\text{Rb} \) it is 3035.7 MHz, for \( ^{87}\text{Rb} \) it is 6834.7 MHz. It is approximately proportional to the nuclear moment and to the difference in \( I \cdot J \) for \( I + 1/2 \) and \( I - 1/2 \), divided by the \( I \) value. Thus

\[
\frac{(\Delta \nu)_{3/2}}{(\Delta \nu)_{5/2}} = 2.2514 \text{ experimentally and we calculate } \frac{(\mu/I)_{3/2}}{(\mu/I)_{5/2}} = \frac{2.7506/1.5}{1.3527/2.5} = 3.3890.
\]

To calculate \( I \cdot J \), we note that squaring both sides of \( \vec{F} = \vec{I} + \vec{J} \) gives:

\[
F(F + 1) = I(I + 1) + J(J + 1) + 2I \cdot J
\]

since the value of the square of an angular momentum \( A \) is always \( A(A+1) \). For \( I = 5/2 \), \( F = 3 \) and 2, the equation gives \( I \cdot J \) values of 5/4 and -7/4 respectively, differing by 3. For \( I = 3/2 \), \( F = 2 \) and 1, the values of \( I \cdot J \) are 3/4 and -5/4, differing by 2. We further get 3.3890 \( \times 2/3 = 2.259 \), which is nearly equal to the ratio 2.2514 of the \( \Delta \nu \) values. (Table 1 and Figure 1 are from the review article “Optical Pumping”, in the April 1972 Reviews of Modern Physics, by Columbia Professor William Happer - Revs. Mod. Phys. 44, 169 - his Table X and Fig. 11.)

In a weak external magnetic field, usually designated \( \vec{H} \) or \( \vec{H}_0 \) in Gauss, there is an extra Zeeman interaction \( -\vec{\mu} \cdot \vec{H} \) with the net atomic magnetic moment \( \vec{\mu} \). In “weak field”, where \( |\vec{\mu} \cdot \vec{H}| \ll \Delta w \) (the hyperfine splitting of the \( F = I+1/2 \) and \( F = I-1/2 \) levels), \( F \) tends to remain a “good quantum number” and \( \vec{\mu} = \vec{\mu}_F \), with the magnetic interaction equal to \( -m_F \mu_B \vec{H} \). Here \( m_F = F, (F-1),...,(-F) \) is the component of \( \vec{F} \) parallel to \( \vec{H} \) and \( \vec{\mu}_F \equiv g_F \mu_B \vec{F} \) defines \( g_F \). In “strong fields”, where \( -\vec{\mu} \cdot \vec{H} \) is large compared with the...
hyperfine splitting, $I$ and $J$ tend to become “uncoupled”, so $F$ is no longer a “good quantum number”, (Paschen-Back region) and $-\mu \cdot \vec{H} = -(m_J g_J + m_I g_I) \mu_B H$, where $m_J$ and $m_I$ are the components of $J$ and $I$ parallel to $\vec{H}$. $m_J$ can take values from $+J$ to $-J$, and $m_I$ from $+I$ to $-I$. Since $|g_J|$ is about $10^3$ times larger than $|g_I|$, the main contribution to $-\mu \cdot \vec{H}$ is from the electron component $\vec{J}$. When $F$ is a good quantum number (weak field), one pictures $\vec{I}$ and $\vec{J}$ as precessing about $\vec{F}$, and $\vec{F}$ as precessing about $\vec{H}$.

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Fig. 1. Magnetic sublevels of the $^{85}$Rb atom. The energy splitting between a pair of excited state sublevels becomes equal to the energy splitting between a pair of ground-state sublevels at a magnetic field of 320 G [from (Bu17a)].
The interaction term is:

$$- \bar{\mu} \cdot \bar{H} = -\mu_{B} \left( g_{J} \bar{J} \cdot \bar{H} + g_{I} \bar{I} \cdot \bar{H} \right),$$

where

$$\bar{J} \cdot \bar{H} = \left( \bar{J} \cdot \bar{F} \right) \left( \bar{F} \cdot \bar{H} \right) / \bar{F}^{2} = \left( m_{F} / F(F+1) \right) \left( \bar{J} \cdot \bar{F} \right) \bar{H}$$

and $\bar{J} \cdot \bar{F}$ is obtained from $(\bar{F} - \bar{J})^{2} = \bar{I}^{2}$, etc. Thus,

$$- \bar{\mu} \cdot \bar{H} = -m_{F} \mu_{B} H \left[ \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} \right] g_{J}$$

$$+ \left[ \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)} \right] g_{I}.$$ 

For $I = 5/2$, $J = 1/2$ it is $(g_{J} = 2.95 \times 10^{-4}$ for $^{85}$Rb)

$$- \bar{\mu} \cdot \bar{H} = -m_{F} \mu_{B} H \left[ \frac{g_{J}}{6} + \frac{5g_{J}}{6} \right] \text{ for } F = 3$$

$$- \bar{\mu} \cdot \bar{H} = -m_{F} \mu_{B} H \left[ \frac{-g_{J}}{6} + \frac{7g_{J}}{6} \right] \text{ for } F = 2.$$ 

For $I = 3/2$, $J = 1/2$ it is $(g_{J} = 0.9987 \times 10^{-3}$ for $^{87}$Rb)

$$- \bar{\mu} \cdot \bar{H} = -m_{F} \mu_{B} H \left[ \frac{g_{J}}{4} + \frac{3g_{J}}{4} \right] \text{ for } F = 2$$

$$- \bar{\mu} \cdot \bar{H} = -m_{F} \mu_{B} H \left[ \frac{-g_{J}}{4} + \frac{5g_{J}}{4} \right] \text{ for } F = 1.$$ 

Problem: Verify these equations.

To the extent that the $g_{J}$ contribution dominates $g_{F}$, note that $|\Delta E|$ for $\Delta m = \pm 1$ is the same for $F=I\pm1/2$ for each $I$, but $\Delta E$ has opposite signs for $F = I + 1/2$ and $F = I - 1/2$. For the $^{2}S_{1/2}$ state, $g_{J}$ is due to the electron spin and is nominally $= -2$ in Dirac theory, while it is -1 for the electron orbital angular momentum (in units of $\mu_{B}$). When quantum electrodynamics (Q.E.D.) corrections are included, $(-g_{S}) = 2(1 + E) \approx 2.0023$ where

$E = \alpha / 2\pi - 0.32848(\alpha / \pi)^{2} + (1.49 \pm 0.02)(\alpha / \pi)^{3}$

to this order of approximation and

$\alpha = e^{2} / \hbar C$ is the fine structure constant: $\alpha \approx 1/137$. A review article, “Experimental determinations of the hyperfine structure in the alkali atoms”, by E. Arimondo, M. Iguscio and P. Violino, Revs. of Mod. Phys. 49, 31-76, Jan. 1977, provides a useful reference on the subject. A discussion of the electron and muon anomalous spin $g$ factors is given in the preceding article (the same issue of R. M. P.).

For arbitrary $\bar{H}$, the Breit-Rabi formula applies. Relative to the midpoint energy for $F=I + 1/2$ and $I - 1/2$ its form depends on sign conventions for $g_{J}$ and $g_{F}$. We choose here
signs for $g$ that are the same as for the moments, contrary to other frequent usage. The
Breit-Rabi formula is written as:

$$w(F = I \pm 1/2, m) = -\mu_B g_J m H \pm \frac{\Delta w}{2} \left[ 1 - \frac{4mx}{(2I+1)} + x^2 \right]^{1/2}$$

where $x = \left( \frac{g_J - g_I}{\Delta w} \right) \mu_B H$ is negative (for $H$ positive) and the $\pm$ are for $I + 1/2$ and $I - 1/2$, respectively.

For $m = \pm(I + 1/2)$, the square root factor is taken as $(1 \mp x)$, respectively, for $|x| < 1$.

The weak field condition is $|x| << 1$, where the last term becomes $\pm \Delta w / 2 \mp \frac{mx\Delta w}{(2I+1)}$ or

$$w = \pm \frac{\Delta w}{2} + m\mu_B H \left[ \frac{\mp g_J}{2I+1} \pm \frac{g_I}{2I+1} - g_J \right],$$

which agrees with the previous expressions.

Note that the value of $F$ does not enter into the square root term except for its sign and the maximum (m) that can be used. (Here we choose $g_J$ for the electron as negative, and $g_I$ (usually) as positive.) The literature often chooses $g_J$ as positive, with confusion of sign for $g_I$ in the expressions. Thus $(g_J - g_I)$ and $x$ in the Breit-Rabi formula are both intrinsically negative. The next approximation in expanding the square root

$$\left[ 1 - \frac{4mx}{(2I+1)} + x^2 \right]^{1/2}$$

involves terms in $x^2$ and $m^2x^2$. In the experiment we set $\tilde{H}$ at various values corresponding to $|x| << 1$ and cause transitions between various $m$ states of the $2S_{1/2}$ ground state system by feeding R. F. signals to the system. The selection rule $\Delta m=\pm 1$, $\Delta F=0$ for such transitions means that we are interested in the frequency for such $\Delta m=1$ transitions (same $I, J, F$). To first approximation, where only terms linear in $x$ are included (weak field), the frequencies are all essentially the same for a given Rb isotope and a given magnetic field $H$. At higher $H$, but $|x|$ still $<< 1$, the quadratic $m^2x^2$ introduces a frequency “splitting” dependent on $m$. The ratio of the splitting to the mean frequency is proportional to $H$. Table 2 lists values of

$$\left[ \left( 1 - 4mx / (2I+1) + x^2 \right)^{1/2} - \left( 1 - 4m'x / (2I+1) + x^2 \right)^{1/2} \right] / |x|$$

for a range of values of $|x|$, for $I = 5/2$ and $3/2$. The sum of the values over possible $m$ values for $F=I+1/2$ always adds to 2.0. At strong fields, $|x| \geq 1$, the $F=I+1/2$ energies all go with $m_f=1/2$ and the $F=I-1/2$ with $m_f=-1/2$, except for the $m_{-1/2}$ state which goes with the $m_f=-1/2$. Since the $m_f$ contribution dominates, this explains the splitting mode seen in Fig. 1. At larger $|x|$, the $m=-(I-1/2)$ to $m=-(I+1/2)$ transition has much higher frequency than the others. Since $f_{max} \approx 50$MHz $<< \Delta \nu$ here, $|x_{max}| \approx 0.01$. 

III. Equipment and Theory for this Optical Pumping Experiment.

Except for peripheral necessary equipment, the main components for this experiment are placed inside the hollow cylindrical volume of a multiple solenoid about 74 cm long and ≈25 cm diameter with horizontal axis, aligned approximately north-south. There are two main sets of solenoid windings both 23 cm in diameter and 74 cm long. One winding is energized by a D.C. current regulated power supply (polarity reversible). It has 1164 turns. The second has 1940 turns and is energized by a saw-tooth shaped voltage-current, which follows the horizontal deflection sweep voltage of the viewing oscilloscope and which is also polarity reversible and amplitude variable. The cylindrical shell has a mu-metal outer magnetic shield and inner soft iron shields to shield partially against the earth’s magnetic field vertical and N-S components and stray fields due to external equipment. These solenoids produce the “external field \( \vec{H}_0 \)“ at the position of the glass bulb containing Rb vapor, which is near the center inside the solenoid. The D.C. current establishes the mean field, while the sweep field varies \( H_0 \) about its mean value in synchronization with the oscilloscope sweep. It may either increase or decrease \( H_0 \) during the sweep. On one (axial) side of the Rb vapor bulb is the light source consisting of a commercial Rb arc lamp, driven by a small external power supply (which should provide no more than 25 mA current). Following the lamp is a cylindrical unit, of about 4 inches diameter, containing a special interference filter, which absorbs the Rb D2 line light but passes the D1 light and some visible light. It also has a pair of plano-convex focusing lenses, followed by a linear polarizer and a quarter wave plate to produce circularly polarized light incident axially on the Rb vapor absorption bulb for optical pumping.

At the end of the solenoid furthest from the Rb lamp, a large positive lens is positioned to focus light on a detector approximately 12 inches further away. The detector is a silicon photo diode about 1 inch in diameter and has an amplifying circuit using a Texas Instruments TL-082 dual R.E.T. Op-AMP. The output is fed to the vertical deflection input of the viewing oscilloscope.

Suppose that the axial magnetic field \( \vec{H}_0 \) is parallel to the light direction and “right circularly polarized light” is produced. For right circularly polarized light, the photon \( \vec{E} \) vector rotates clockwise as viewed looking at the incident light. The photon angular momentum is then \( (−\hbar) \) relative to its direction of motion. (For left circularly polarized light it is \( (+\hbar) \)). The photon angular momentum relative to the applied field \( \vec{H}_0 \) direction depends on whether \( \vec{H}_0 \) is in the same or opposite direction as the light flow.

Consider the situation where the photon angular momentum is \((+1)\) relative to \( \vec{H}_0 \). We can call this \( \sigma^+ \) light. The Rb atoms are initially nearly equally distributed among the \( ^2S_{1/2}, F=\frac{1}{2} \) and \( F=\frac{3}{2} \), various \( m_F \) states, because their energy separation is \( << kT \) (the thermal energy) and the Boltzman factor \( e^{-\epsilon/kT} \) is nearly the same for all of the \( ^2S_{1/2} \) states near or above room temperature. When a \( \sigma^+ \) D1 photon is absorbed, the electric dipole
selection rule is $|\Delta\ell| = 1, |\Delta l| = 0, \pm 1, |\Delta F| = 0, \pm 1$ and, most important, $\Delta m_F = +1$ (It would be $\Delta m_F = -1$ for a $(\sigma\sigma)\text{-photon}$). The 5P state then decays back to the $5^2S_{1/2}$ ground state by photon emission, with $\Delta m_F = 0, \pm 1$. Since the pumping is unidirectional for $\Delta m_F$ during absorption, eventually most of the atoms reach the highest $F, m_F = I + 1/2$ $^2P_{1/2}$ excited level on absorption and eventually reach the $m_F = F = I + 1/2$ ground $^2S_{1/2}$ level. Then the Rb vapor in the bulb becomes transparent to $\sigma^+$ Rb D1 radiation, since there is no $m > I + 1/2$ $^2P_{1/2}$ excited state to go to. The “population” is then said to be “optically pumped” to a single ground $m$ state. For $(\sigma\sigma)$ pumping light the atoms similarly tend to be pumped to the $^2S_{1/2}, m_F = -(I + 1/2)$ state. This discussion uses the $\sigma^+, \sigma-$ convention relative to the direction of $\tilde{H}_o$. If, during the oscilloscope sweep time, the net axial magnetic field reverses sign, there is always enough (small) $\tilde{H}_o$ perpendicular to the axis to have the pumped $m$ follow $\tilde{H}_o$. During the axial $\tilde{H}_o$ reversal the photon $\sigma^+$ or $\sigma-$ reverses sign and the atoms are in the opposite extreme $m$ furthest from the fully pumped condition - so there is a strong absorption and a longer pumping time is required to reach near transparency. This is observed as a pulse (up or down) on the oscilloscope during the period of lower light transmission. (For a scattered light detector to the side of the Rb bulb, it would also show as a pulse due to increased re-radiated light following absorption.)

For $\tilde{H}_o \neq 0$, spin flip is induced by R.F. fields perpendicular to $\tilde{H}_o$. The few turn R.F. coils about the Rb bulb, having axes perpendicular to the large cylindrical tube, are fed by sinusoidal signals ranging from audio frequencies to $\leq 50$MHz. A large General Radio R.F. signal generator supplies the R.F. The principle involved is as follows. The atomic magnetic moment $\vec{\mu}$ precesses about the external field $\tilde{H}_o$ with its precession frequency $\omega$ as discussed in Part II. A sinusoidal varying $\tilde{H}_i$ perpendicular to $\tilde{H}_o$ can be regarded as the (vector) sum of two fields each of strength $\left|\tilde{H}_i / 2\right|$ precessing with the R.F. frequency in opposite senses about $\tilde{H}_o$, each at R.F. frequency $\omega_1$. If $\omega_1\hbar$ equals the transition energy for $|\Delta m| = 1$ it can induce such a transition. The component rotating in the same direction as $\vec{\mu}$ about $\tilde{H}_o$, if $90^\circ$ leading or following $\vec{\mu}$ in its precession, provides a continuous torque either tending to increase or decrease the angle of $\vec{\mu}$ or $\vec{F}$ with $\tilde{H}_o$, and thus change $m$ by $\pm 1$. Since $\tilde{H}_o$ has the oscilloscope sweep synchronized time variation about its mean value, if $\tilde{H}_o$ coincides with such a condition during the sweep, it is seen as a pulse on the oscilloscope, since it always tends to increase the occupation of states away from the fully pumped state. As when $H_{axial}$ passes through zero, the increased absorption is followed by a recovery of time duration $10^{-2}$ to $10^{-3}$ sec, the mean time for the “pumping” to be effective. A similar recovery follows the R.F. spin flip pulse rise.

Since the Rb vapor pressure increases nearly exponentially with temperature, the bulb must be at optimum temperature ($55^\circ$ to $60^\circ$C) for this part. If the fully pumped state corresponds to $\pm (I + 1/2)$, one might expect that only transitions from this state would
be effective for pulse production. However, pumping is incomplete, so other states are partially occupied, in decreasing order from the fully pumped state, and the full array of possible R.F. transitions to less populated states may be observed as pulses of decreasing size for ones further from the fully pumped state. These pulses are all very small compared to that where \( \vec{H}_{\text{axial}} \) passes through zero, which causes a reversal of the effective pumping light \((\sigma^+) \rightarrow (\sigma^-) \) or \((\sigma^-) \rightarrow (\sigma^+) \) relative to \( \vec{H}_0 \) that reverses.

If the R.F. field is not small, two or more quantum transitions, \(|\Delta m|= 2, 3, ..., \), may be seen. Also, second and third harmonies in the R.F. are large enough to permit observations of the R.F. transitions at half or a third the fundamental frequency for the transitions. For \(^{87}\text{Rb} \ (I = 3/2) \) the fundamental frequency is approximately 14 MHz/Amp. For the less abundant \(^{85}\text{Rb} \ (I = 5/2) \) the frequencies are about 2/3 as large. If a frequency \( f_1 \) is applied by the main R.F. coil, and \( f_2 \sim 200 \text{ kHz} \) is applied to the coils with axis parallel to the main field, transitions are seen when \( f_1, (f_1 + f_2), \) and \( (f_1 - f_2) \) are at the correct frequency (beat frequency terms).

The R.F. frequency is read using a digital frequency meter. The regulated current supply for the D.C. axial \( H \) is read using a 0.01 ohm precision shunt (good to 0.01\%). The voltage across it is read by a 5½ digit multimeter, which has \( \approx 0.04\% \) accuracy ± 1 digit in the last place. Since resonance frequency shifts are proportional to current shifts, care should be taken to keep constant current readings while measuring the resonance frequency for the various transitions at a given current. The power supply for the Rb lamp has a knob having off-standby-on positions. Allow it to warm up \( \geq 1 \) minute in the standby position before switching to “on”. The current is read on a separate meter and should not exceed 25 mA (full scale). A current \( \approx 23 \text{ mA} \) is suggested for operation.

**Calculations to be made before the first lab**

Show that the axial magnetic field \( B \) for the main solenoid, 23 cm diameter, 74 cm long, 1164 turns would be \( B = 18.88 \ I \) (gauss and amps) if the ferro magnetic shielding was absent. The mu metal tends to reduce the return path reluctance and give a \( B \) vs. \( I \) relation closer to that for an infinite length solenoid of the same turns/cm. Show that this is \( B = 19.77 \ I \). Empirically, a relation \( B \approx 19.53 \ I \) seems to apply, where the multiplying factor has a weak \( I \) dependence.

Using \( B = 19.53 \ I \) and \( f \approx \left| g_J \right| \frac{\mu_B H}{2I+1} \) (for \( \Delta m = \pm 1 \) as shown near the top of page 3 and 4) calculate \( f/I \) in MHz/Amp for the 99% present \(^{87}\text{Rb} \ (I = 3/2) \) and for the \( \sim 1\% \) abundant \(^{85}\text{Rb} \ (I = 5/2) \). The values will help you locate the Zeeman mean frequencies vs. solenoid current. For \( I \sim 0.5 \text{ amp} \) where the line splitting is small, so the Zeeman effect applies, if one assumes that \( \left| g_J \right| \frac{\mu_B}{2I+1} \) is known, then \( H \) is determined from the frequency and this may be used to establish the \( B/I \) coefficient. (Note \( B \equiv H \) here.)
First Week (continued the second week, etc.)
The student should read these instructions and do the above calculations before the first lab. The instructor will point out features of the apparatus, with necessary precautions. After familiarizing with the equipment with the instructor’s help, start the bulb heating, turn on the detector and the Rb lamp. The linear sweep $H$ field uses the $> 100$ V high impedance oscilloscope sweep saw tooth wave form, fed to a transistor circuit which converts it to a low impedance few volt amplitude saw tooth voltage having zero near its mean. It has a 50 ohm resistor in series to the output, plus a decade resistance box $R$ having 10 ohm steps. The coil resistance is 10 ohms, so the magnitude of the saw tooth current sweep can be varied over a factor $\sim 3$ using the decade resistor box (inversely as 50 ohms + $R$).

With zero main field current to the solenoid, you should observe pulses of 2 to 4 volts at $50^\circ - 60^\circ$ C when $H_{axial}$ passes through zero. The pulse should be observed for a range of sweep speeds from about 1 msec/cm to 500 msec/cm. Note that there is some sweep speed dependence on the horizontal position of the build up (front edge) of the pulse. Note the recovery time constant due to the optical pumping.

When taking data using the R.F. for transitions, a sweep rate of about 2 seconds per sweep should be used. This involves non-zero D.C. axial fields. The horizontal sweep positioning is done as follows. Using the current reversing switch one way, change to the midway open position. The residual iron-mu-metal shielding for zero current is a small amount in the sense of the preceding field direction. Note the position of the front edge of the zero field pulse and set to the middle of the oscilloscope face (left-right center). Now use reversed current followed by zero current for reversed residual magnetization. There should be a small sidewise shift in the pulse position. Repeat and set the horizontal positioning so the pulses for opposite residual magnetization are equidistant on opposite sides of left-right center of the display. It is best to use the series decade box at maximum for small $H_o$ and at minimum for higher $H_o$ (main field current).

Now see what (small) values of the D.C. coil current are required to shift the peaks 4 cm to the left and right. (This calibrates the magnitude of the saw tooth $H$ sweep in terms of the main field current.) Repeat with the decade resistance at minimum.

When the bulb temperature is in the $55^\circ - 60^\circ$ C range, you can investigate the R.F. transition frequencies for $I = 0.5, 1.0, 2.0, 3.0, 3.5$ and 4 amps. Best results seem to be obtained when the sweep field-reversing switch puts the largest pulse to the right of the weaker pulses. For one main field direction pumping is to maximum $m=(I+1/2)$ and the $m = (I + ½)$ to $(I - ½)$ transition is largest. For the opposite main field direction, pumping is to $m = -(I + ½)$ and the large pulse is for the $-(I + ½)$ to $-(I - ½)$ transition (highest frequency). When viewing the transitions, start with maximum R.F. voltage for each pulse and then reduce the R.F. drive to narrow the pulse, but not so much that it is no longer visible. Try to measure for all six $^{87}$Rb transitions and as many $^{85}$Rb transitions as you can. (Check the current and re-adjust frequently (if necessary) for best results.) The oscilloscope gain should be increased for the very weak pulses and for $^{85}$Rb. With $I = 0.5$ Amp, connect the audio oscillator to the coils having axis parallel to the main field.
For 100 kHz and 200 kHz note that resonances are obtained with the main R.F., as before, and also at the sum and difference beat frequencies equal to the resonance frequency. Before the second week, use the Breit-Rabi formula (as below) to calculate the expected transition frequencies for the range of currents that you will use.

Analysis
Calculate the expected frequencies using the Breit-Rabi formula.

For $^{87}\text{Rb}$ ($I = 3/2$):

$$|x| = 4.10236 \cdot 10^{-4} H$$

$$f(\text{MHz}) = 3417.34 \left( \left[ 1 + m \frac{|x| + x^2}{1.5} \right]^{1/2} - \left[ 1 + (m-1) \frac{|x| + x^2}{1.5} \right]^{1/2} \right)$$

$\mp 0.0013978H$ (for $F = I \pm 1/2$)

For $^{85}\text{Rb}$ ($I = 5/2$):

$$|x| = 9.2302 \cdot 10^{-4} H$$

$$f(\text{MHz}) = 1517.75 \left( \left[ 1 + \frac{m |x| + x^2}{1.5} \right]^{1/2} - \left[ 1 + \frac{(m-1) |x| + x^2}{1.5} \right]^{1/2} \right)$$

$\mp 4.1245 \cdot 10^{-4} H$ (for $F = I \pm 1/2$)

You should calculate the theoretical frequencies say for $I = 3$ Amps (a current to be used during the first week) using $H = 19.53 I$. If it seems that a slightly different factor than 19.53 is needed use that factor for subsequent calculations.

In comparing the calculated and measured frequencies for each current, a tiny systematic difference is not important. The differences of each transition frequency from that of the lowest frequency (highest $m$) transition for that current are sensitive to a small systematic shift between the observed and calculated values and should be emphasized. This also helps you to correlate the observed frequencies with the various $F = I \pm 1/2$ transitions, six for $^{85}\text{Rb}$ and ten for $^{87}\text{Rb}$. Do the approximate comparisons.

The value of $g_i$ is obtained by noting the frequency difference between the same $m_i \rightarrow m_j$ for $F = I - 1/2$ and $F = I + 1/2$. Obtain $g_i$ in nuclear magnetons and compare with the values in Table 1. (also for $^{85}\text{Rb}$ if possible).

The values of $I$ for the isotopes are established either by the number of transitions at each field, either by the approximate relation between $f$ and $H$. Note that only zero integer or odd half integer values for $I$ are possible. To see the approximate $m$ dependence of transition frequencies, the Breit–Rabi formula must be expanded to terms in $m^2 x^2$. Ignoring the last term:
$w(F = I \pm 1/2, m) = \pm \frac{\Delta w}{2} \left[ 1 + \frac{4m|x|}{2I + 1} + x^2 \right]^{1/2}$. 

Using $(1 + \varepsilon)^{1/2} \approx 1 + \varepsilon / 2 - \varepsilon^2 / 8$ and keeping only the small terms that contain $m$:

$$|w(m)| \approx \frac{\Delta w}{2} \left[ 1 + \frac{2m|x|}{2I + 1} - 2 \left( \frac{m|x|}{2I + 1} \right)^2 \right],$$

$$f(m) = |w(m) - w(m - 1)| \approx \frac{\Delta w}{2} \left[ \frac{2|x|}{2I + 1} - 2(2m - 1) \left( \frac{|x|^2}{2I + 1} \right) \right].$$

The part $\frac{\Delta w|x|}{2I + 1} = f_{\text{mean}}$ is essentially the mean frequency for the $m = 1$ to $m = 0$ and $m = 0$ to $m = -1$ transitions for $F = I + 1/2$ and $F = I - 1/2$. In the Breit-Rabi formula

$$x = \frac{(g_j - g_i) \mu_B H}{\Delta w},$$

so that

$$\frac{\Delta w|x|}{2I + 1} = \frac{(g_j - g_i) \mu_B H}{2I + 1} = f_{\text{mean}}.$$Thus $f_{\text{mean}}$ is proportional to $\frac{H}{2I + 1}$.

Consider $|f(m_1) - f(m_2)| = \Delta w \left| 2(m_1 - m_2) \left[ \frac{x}{2I + 1} \right]^2 \right|$. For $m_1 = I + 1/2$ and $m_2 = -(I - 1/2)$ we obtain $(m_1 - m_2) = 2I$ and

$$|f(m_1) - f(m_2)| = 4I \Delta w \left[ \frac{x}{2I + 1} \right]^2.$$

Using $|x| = (2I + 1) \frac{f_{\text{mean}}}{\Delta w}$ gives: $|f(m_1) - f(m_2)| \approx \frac{4I}{\Delta w} (f_{\text{mean}})^2$, or $\Delta w \approx \frac{4I (f_{\text{mean}})^2}{f(m_1) - f(m_2)}$.

This permits an approximate evaluation of the $\Delta w$ obtained only from the frequencies of the biggest peaks for each switch direction. A more precise evaluation is made if you assume that the values of the known $\Delta w$ used in the expressions to calculate $f$ for $^{87}$Rb and $^{85}$Rb may not be in the best agreement with your data (for high currents). If one increases $\Delta w$ by 5%, the values of $|x|$ for a given current are multiplied by $(1.05)^{-1}$ and the factor multiplying the main term containing $|x|$ and $m$ is multiplied by $(1.05)^{1}$. This decreases $|f(m_1) - f(m_2)|$ by multiplying by $(1.05)^{-1}$. Thus by taking the ratio of observed to calculated $|f(m_1) - f(m_2)|$ one divides the reference $\Delta w$ by this factor to obtain your measured $\Delta w$. Since the difference in frequency for extreme $m$ values increases essentially quadratically with current, the evaluation is best made using the highest current data where the deviation from simple Zeeman splitting is largest. You should probably obtain $\Delta w$ to significantly less than 1% uncertainty.