Leaving the Large Radius limit: Racetrack Inflation

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July 1, 2005

Abstract

After many years of waiting for more data, particle physics is getting a new boost from both the observational side with cosmology [WMAP, Planck] and from collider experiments [LHC]. With the coming of this new phase we need to prepare ourselves as theorists and ....???. In recent years there has been a notable move towards finding cosmological features within a string theoretic framework []). It is our belief that this process should proceed in both directions. If we genuinely hope to explain the features of our universe, in particular its cosmological evolution, within string theory, then we should try to use cosmology to constrain the string theory that we look at. In [RI], based on [kklt] the authors find an example of inflation in string theory where all the moduli are stabilised. If we are to take such proposals seriously we should look closely at their region of validity. Our modest goal here is to do this for the case of the aforementioned work [RI]. In particular, the [RI] work ...
Contents
1 Introduction

NEEDS WORK In recent years there have been many attempts at finding a model of inflation from string theory. The possibility of making connections with cosmology is exciting both from both perspectives. In particular Inflation is a successful idea that is only realized at the level of phenomenology. It needs fundamental theory to provide a natural potential and an inflaton field. Likewise string theory is searching for an opportunity to prove itself in the experimental arena. The possibility that inflation could be a bridge between string theory and cosmology is an exciting one and hence a lot of effort has gone into making such ideas work. As many authors have realised [], a study of inflation from string theory cannot be separated from the issue of moduli stabilisation. DETAIL here The question of what string theory could say about cosmology is a tantalising one, but not the focus of this work. The purpose of this work is somewhat reversed. We ask what cosmology could say about string theory. In particular, given a model (Racetrack inflation) from string theory that provides inflation, we invert the situation and ask the following question. Assuming we want inflation to work, within the bounds of this model, can we say something about the compactification manifold?

We make no arguments about the naturalness of these models and we do not advocate for or against them. We simply use them to ask questions that we find interesting.

In the interests of being self contained we will start off with a brief review of the features that go into this model. To that end, we describe the kklt scenario.

2 A Review of the KKLT Picture

We start off with a brief review of the KKLT scenario []. We work in type IIB string theory compactified on a Calabi-Yau with nontrivial NS and RR three-form fluxes. By including quantum corrections and extended objects; branes in string theory they explain how to stabilise all the moduli (Kähler structure moduli, complex structure moduli and the dilaton) and find a metastable deSitter solution. As described in [] in these constructions one can stabilise the complex structure moduli but not the Kähler moduli due to the no-scale structure of the low energy 4D $N = 1$ supergravity potential

$$ V_F = e^K \left( \sum_{a,b} g^{ab} D_a W D_b W - 3|W|^2 \right) $$

where $a, b$ run over all moduli fields, $K$ is the Kähler potential and $W$ is a holomorphic function of the moduli called the superpotential. We study the case of only one Kähler modulus $T^1$. In this expression $g^{ab}$ is the matrix inverse of the Kähler metric $g_{ab} = \partial_a \partial_b K$ and $D_a W = \partial_a W + \partial_a K W$. The explicit form for $K$ and $W$ can be found semiclassically using the symmetries of the underlying 10-d theory [Witten 1985]. This can be derived without any knowledge of the specific CY. To leading order in $\alpha'$ and $g_s$ we find [GVW, KKLT]

\footnote{discuss the validity?}
\[ W = \int G_3 \wedge \Omega = W_0 \quad \text{and} \quad K = -3 \ln(T + \overline{T}) - \ln(-i(\tau - \overline{\tau}) - \ln [-i \int \Omega \wedge \overline{\Omega}] \quad (2) \]

where \( W_0 \) is independent of \( T \), the single Kähler modulus, \( \tau \) is the axiodilaton and \( G_3 = F_3 - \tau H_3 \) where \( F \) and \( H \) are the 3-form fluxes. Putting this form into the potential we see that the Kähler modulus \( T \) cancels the \( 3|W|^2 \) term and hence drops out of the potential, leaving the positive semi-definite potential characteristic of no-scale models.

\[ V = e^K \left( \sum_{i,j} g^{ij} D_i W D_j \overline{W} \right) \quad (3) \]

where \( i,j \) now run only over the complex structure moduli and the dilaton which we can stabilise using this potential. To stabilise the Kähler modulus \( T \) we need to lift the remaining flat direction. This can be done by adding non-perturbative effects to induce a nontrivial superpotential and hence a potential for \( T \). In this way all the moduli are imagined to be stabilised. We then break supersymmetry and raise the AdS min to a dS min by adding a stack of D branes which contributes to the potential in the following way.

Our analysis is based on racetrack inflation which is built on the KKLT scenario but differs only in its choice of nonperturbative superpotential. In the racetrack picture the superpotential takes the modified racetrack form [RI]

\[ W = W_0 + A e^{-aT} + B e^{-bT} \quad (4) \]

such as would be obtained from a gaugino condensate in a theory with a product (semi-simple) gauge group. [INCLUDE review of this in appendix??] In the case where the group is \( SU(N) \times SU(M) \) we find \( a = \frac{2N}{N+M} \), \( b = \frac{2M}{N+M} \). The KKLT superpotential is the special case where \( AB = 0 \), and the standard racetrack potential is the case where \( W_0 = 0 \) which has been discussed in order to fix the dilaton at weak coupling in the heterotic string [].

### 3 Breaking the no-scale Structure

We noted earlier the no-scale structure of the supergravity potential. This is only true to first order for the Kähler potential. We mentioned above how non-perturbative corrections to the superpotential break the no scale structure and generate a potential for \( T \). One might wonder why we would expect relatively small contributions to play an important role. Of course, this is the case here as we are adding corrections on top of an exactly flat potential. We could, however, break the no-scale structure in other ways. We expect and find, as did [?], that the \( \alpha' \) corrections break the no-scale structure and generate a correction to the supergravity potential that is proportional to the Euler number of the internal manifold. We start by showing how this happens.

We insert the corrected Kähler potential and its metric, \( g_{T \overline{T}} = \frac{(8X^3 + L^2)}{6A(8X^3 - 2L)} \) into the supergravity potential, where we have used \( T = X + iY \) and \( L \) is the contribution from the loop term [see .
\[ V_F = e^K \left( \sum_{i,j} g^{ij} D_i W D_j W + g^{T} D_T W D_T W - 3|W|^2 \right) \] (5)

where \( i, j \) run over the complex structure moduli and the dilaton. At leading order the final 2 terms in the potential would cancel, but we see that this is no longer the case and we generate a potential for the volume modulus, \( T \). We start by looking at the simplest case where the superpotential does not receive nonperturbative corrections to ensure that any effect we may see is due to corrections to the Kahler potential only. Indeed, by breaking the no-scale structure we will generate a potential for \( T \) that may allow us to stabilise \( T \) and make the nonperturbative terms unnecessary, so we start by checking this. Putting in \( W = W_0 \) gives

\[ V_T = \frac{6LW_0^2}{(8X^3 + L)(8X^3 - 2L)} \] (6)

We see that the potential is exactly flat in the Y direction. As expected the \( L = 0 \) limit gives us the expected \( V_T = 0 \) result. We look for turning points of this potential and hope that there will be a minimum to stabilise \( T \).

\[ \frac{dV}{dX} = \frac{-36(64X^5 - 4LX^2)LW_0^2}{((8X^3 + L)(8X^3 - 2L))^2} = 0 \] (7)

\[ \rightarrow 4X^2(16X^3 - L) = 0 \] (8)

\[ \rightarrow X_{\text{max}} = \frac{3}{\sqrt[3]{16}} \] (9)

Alas, we find that there are no minima and only one maximum in the potential at \( X = \frac{3}{\sqrt[3]{16}} \). We could see this explicitly by looking at the potential below. The lesson here is that we cannot stabilize the volume by simply adding \( \alpha' \) corrections as was the hope of [?]. [ Also could we consider the asymptote stable?]

Figure 1: \( V \) for \( W = W_0 = -1/25000 \)

5
4 Review of Racetrack Inflation

We have seen that simply finding a potential for the volume $T$ is not enough. We need to find a potential with a minimum to stabilise $T$ and more promisingly a potential with a flat enough region along which inflation can occur. We return now to the methods of KKLT \[^2\] where the potential is lifted by adding nonperturbative corrections to the superpotential. We might expect to find inflation in some region of this potential. This turns out not to be possible as the potentials are not sufficiently flat. At this point we have a few options, we can add stacks of branes to various regions on the compact manifold and attempt to get brane inflation \[^2\] [add DBI paper and tye] or we could try a little harder to get modular inflation from nonperturbative corrections. It was found \[^2\] that it is possible to get string inflation from a geometrical modulus without introducing interacting D-branes. By adding nonperturbative potentials of the modified racetrack type it is possible to find saddle point regions and hence slow-roll inflation. In this sense the authors of \[^2\] have revived the old ideas of modular inflation with a flat enough potential. The $\eta$ problem is not a problem here as usually it is the $e^K$ term that introduces the $\eta$ problem. In the case of \[^2\] this problem is avoided as the inflaton will be $Y$, the Imaginary part of $T$, which does not enter in $e^K$. We will briefly review this work here. For a review of the $\eta$ problem see Appendix or ref?.

As we have said, the nonperturbative superpotential is assumed to have the modified racetrack form.

$$W = W_0 + A e^{-aT} + B e^{-bT}. \quad (11)$$

where $W_0$ is the effective superpotential as a function of all the complex structure moduli and the dilaton that have already been fixed. In the KKLT scenario $W_0$ is required to be small ($W_0 < 10^{-4}$) which is achieved by tuning fluxes. The exponential terms are what we would expect from a gaugino condensation in a theory with a product gauge group. \[^2\] For example, for an $SU(N) \times SU(M)$ group we would have $a = 2\pi/M$ and $b = 2\pi/N$. The scale of $A$ and $B$ is set by the cutoff of the effective theory, so we expect both to be small when expressed in Planck units.\[^38\] in RI. This choice of potential is particularly useful as it is fairly simple to engineer where the potential will have a globally supersymmetric minimum. For the gauge theory the potential will have a globally supersymmetric minimum, $W' = 0$, when

$$T = \frac{NM}{2\pi(M - N)} \log \left(\frac{-MB}{NA}\right) \quad (12)$$

We can then choose $M$ and $N$ to place the minimum at large $T$ or weak coupling. This is the assumption of \[^2\] to ensure that inflation occurs near the minimum and we can trust the large $T$ approximation. The same is true for the the minima of the full supergravity potential. It is this assumption that we want to relax in this work to understand how adding $\alpha'$ corrections to the Kähler potential might change this result. Following KKLT, the scalar potential comes from 2 terms

$$V = V_F + \delta V \quad (13)$$

The first term comes from the standard $\mathcal{N} = 1$ supergravity formula for the F-term potential. In Planck units this is \[^39\] of RI
\[ V_F = e^K \left( \sum_{i,j} g^{ij} D_i W D_j W - 3|W|^2 \right) \] (14)

where \( i, j \) run over all moduli fields. We start by reviewing the results for the large radius \( X \) case. In this case the Kähler potential is

\[ K = -3 \log(T + \bar{T}) \] (15)

and the metric is \( g_{TT} = \frac{3}{(T + \bar{T})^2} \).

The supersymmetric configurations are given by \( D_T W = 0 \) i.e.

\[ 2XW' - 3W = 0 \] (16)

where \( ' = \frac{d}{dT} \). Setting \( T = X + iY \), and substituting into the scalar potential gives

\[ V_F = \frac{e^{-aX}}{6X^2} [ aA^2 (aX + 3) - e^{-aX} + 3W_0 aA \cos(aY)] + \]
\[ + \frac{e^{-bX}}{6X^2} [ bB^2 (bX + 3) - e^{-bX} + 3W_0 bB \cos(bY)] + \]
\[ + \frac{e^{-(a+b)X}}{6X^2} [ AB (2abX + 3a + 3b) \cos((a - b)Y)] \] (17)

which has many turning points. In particular, the above potential has a saddle point at \( Y = 0 \) and at regular intervals as it is periodic in \( Y \). We find that \( X \) can be stabilised near the the KKLT minima and near the saddle point at \( Y = 0 \). We also find that we can get sufficient slow roll near the saddle point.

The second term in the potential, \( \delta V \), is induced by the tension of the anti-D3 branes added \([?]\) to break supersymmetry and lift the potential from an AdS to a dS minimum. The introduction of the anti-branes doesn’t introduce extra translational moduli as their position is fixed by the fluxes. The contribution is positive definite as is of the following form

\[ \delta V = \frac{E}{X^\alpha} \] (18)

where the coefficient \( E \) depends on the the tension of the branes \( T_3 \), the number of branes and the warp factor. For this reason we can discretely tune \( E \) and the supersymmetry breaking in the system but not to arbitrary precision. It is in this way that \([?]\) found a metastable de Sitter solution. Depending on where the anti-branes sit we get different results for the exponent \( \alpha \). If the anti-branes are sitting at the end of the throat in the region of maximum warping we find \( \alpha = 2 \). \( \alpha = 3 \) corresponds to the anti-branes sitting in the unwarped region. Since the former is energetically favoured we take \( \alpha = 2 \) henceforth.

5 Corrected Racetrack

We now do a similar analysis as above but we include the \( \alpha' \) corrections to the Kähler potential, the derivation of which is renegated to Appendix A where we find
\[ K = -\log \left( (T + \mathcal{T})^3 + L \right) \]  
\[ g_{T\mathcal{T}} = \frac{6X(8X^3 - 2L)}{(8X^3 + L)^2} \]  

For supersymmetric configurations \( D_T W = 0 \) i.e.
\[ (8X^3 + L)W' - 12WX^2 = 0 \]  
In the limit where \( L = 0 \) (i.e. no loop term), we get \( 2WX' - 3W = 0 \) as expected. For the scalar potential we find

\[ V = \frac{E}{X^2 (8X^3 + L)(8X^3 - 2L)6X} \left[ A^2(a(8X^3 + L)(a(8X^3 + L) + 24X^2) + \\
+ 36XL) e^{-aX} + 24W_0 AX(3L + aX(8X^3 + L)) \cos(aY) \right] + \\
+ \frac{e^{-bX}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ B^2(b(8X^3 + L)(b(8X^3 + L) + 24X^2) + \\
+ 36XL) e^{-bX} + 24W_0 BX(3L + bX(8X^3 + L)) \cos(bY) \right] + \\
+ \frac{e^{-(a+b)X}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ 2AB((8X^3 + L)((8X^3 + L)ab + \\
+ 12X^2(a + b)) + 36XL) \cos((a - b)Y) \right] \]  

We can check the \( X \gg L \) or equivalently \( L = 0 \) limit and we find the same potential as we found in the previous section as expected. We also plot this function with the same values for the parameters \( a, b, A, B, W_0, E \) as in [?] and find the exact same plots verifying the validity of ignoring the loop corrections for these choice of parameters.

![Figure 2: Racetrack Potential with M= 100, N= 90](image)

We also plot the \( Y = 0 \) slice to show that there is in fact a minimum and so we can avoid decompactification for a suitable range of initial conditions.
I.e plots for $L = 0$ and $L = 2.33$, which is the numerical value of the loop contribution, look identical, when the minimum is at large $X$, in this case $X_{\text{min}} \approx 123.22$. This is expected as we input (by choosing the form of the superpotential) that we are looking in regions where the global minimum will be at large $X$. Remember we said that we could do this by choosing $M$ and $N$ appropriately, using eqn (10). Having chosen $M$ and $N$ appropriately we effectively place the saddle point at large $X$ and hence expect the loop term to be dominated entirely by the $X^3$ terms. It is clear that adding the instanton corrections to the Kähler potential will not change this result as they are exponentially smaller than the loop term. To quantify the effect of the loop term we need to place the minimum at smaller $X$. We can do so by adjusting $M$, $N$ and their relative size as we know from eqn (10).

It is clear that we can’t see the effect of the loop term up here in the large $X$ region. Going to the small $X$ range of the same plot allows us to see the effect of the loop term.

Before looking further at the effect of the loop term we discuss another important feature of the Racetrack Inflation model, a scaling symmetry, that may turn out to be useful.
6 Scaling Properties

We can see quite simply that by rescaling the parameters in the model we can shift the minima while leaving the features of the potential, including the slow roll parameters, unchanged. This simple rescaling is

\[ a \rightarrow \frac{a}{\lambda}, \quad b \rightarrow \frac{b}{\lambda}, \quad E \rightarrow \lambda^2 E \]  

with

\[ A \rightarrow \lambda^{3/2} A, \quad B \rightarrow \lambda^{3/2} B, \quad W_0 \rightarrow \lambda^{3/2} W_0 \]  

Under these rescalings the potential and the slow roll parameters \( \epsilon \) and \( \eta \) do not change if the fields are also rescaled as follows

\[ X \rightarrow \lambda X, \quad Y \rightarrow \lambda Y \]

It is this change that reflects the change in position of the minima. Thus it is a prediction of these models that one can move the saddle point of the potential to an arbitrary position \( X \) and still achieve inflation. We do not expect this to be the case as the assumption going in was that the minima occur at large \( X \) specifically so that the \( \alpha' \) corrections would not play a role. We will use this very scaling to find where this assumption breaks down in section ().

7 Computing Slow Roll Parameters

Before moving on we should illustrate the claim that one can get slow roll inflation near the saddle point for the racetrack potential. To do so we need to calculate the slow roll parameters at the saddle point. For the choice of parameters that we have used (from [?]) we find the saddle point at

\[ X_{\text{saddle}} = 123.22, \quad Y_{\text{saddle}} = 0, \quad V_{\text{saddle}} = 1.655 \times 10^{-16} \]  

We know that at the saddle point \( \epsilon \) is exactly zero since \( \epsilon \frac{V'}{V} \) which is zero at a turning point. Note that the derivative here is with respect to \( Y \) the inflaton. We now calculate \( \eta \)
taking into account that the kinetic term for the inflaton, $Y$, does not have a canonical form. We do this in general so that we can use the form that we find when we look at the corrected potential. Specifically the kinetic term is

$$L_{\text{kin}} = 2g_T T^2 \left( \partial_\mu X \partial^\mu X + \partial_\nu Y \partial^\nu Y \right)$$

(27)

where the factor of 2 is a result of the relation between $X$ and $T$. Taking this normalisation into account changes $\eta = \frac{V''}{V}$ to become $\eta = \frac{V'''}{2g_T T^4}$. For the large radius case, we use $g_T T = \frac{5}{4X^2}$ resulting in $\eta = \frac{2X^2 V''}{3V}$ with $X$ evaluated at the saddle point and we find (in agreement with [?])

$$\eta_{saddle} = -0.0060$$

(28)

We have kept our calculation general so that it is a simple matter to include the effect of the loop term and understand how $\eta$ is affected. Specifically, we input the loop corrected metric $g_T T = \frac{6X(8X^3-2L)}{(8X^3+L)^2}$ and find $\eta$ analytically to be

$$\eta = \frac{1}{2g_T T^2} \frac{V''}{V}$$

$$= \frac{2(8X^3 + L)^2}{6X(8X^3 - 2L)(8X^3 + L)(8X^3 - 2L)6X} \left[ -a^2 24W_0 AX(3L + aX(8X^3 + L)) \cos(aY) \right] +$$

$$+ \frac{e^{-bX}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ -b^2 24W_0 BX(3L + bX(8X^3 + L)) \cos(bY) \right] +$$

$$+ \frac{e^{-(a+b)X}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ -(a - b)^2 2AB \cos((a - b)Y) \right] \left( (8X^3 + L)((8X^3 + L)ab + \right.$$}

$$+ 12X^2(a + b) + 36XL \right]$$

$$\div \left\{ \frac{e^{-aX}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ A^2 (a(8X^3 + L)(a(8X^3 + L) + 24X^2)) + \right.$$}

$$+ 36XL) e^{-aX} + 24W_0 AX(3L + aX(8X^3 + L)) \cos(aY) \right] +$$

$$+ \frac{e^{-bX}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ B^2 (b(8X^3 + L)(b(8X^3 + L) + 24X^2)) + \right.$$}

$$+ 36XL) e^{-bX} + 24W_0 BX(3L + bX(8X^3 + L)) \cos(bY) \right] +$$

$$+ \frac{e^{-(a+b)X}}{(8X^3 + L)(8X^3 - 2L)6X} \left[ 2AB ((8X^3 + L)((8X^3 + L)ab + \right.$$}

$$+ 12X^2(a + b) + 36XL \cos((a - b)Y) \right]\}$$

(29)

Of course it requires numerical simulations to evaluate $\eta$. Also, it is worth noting that the result is very sensitive to changes in any of the parameters. In the following table we have

11
evaluated $\eta$ for a range of values of $\lambda$ and compared our results with or without the loop term. Remember $\lambda$ is the scaling that we use to shift the turning points to smaller $X$. This allows us to see just where in $X$, $\eta$ is no longer much less than one. We can see from the table, that including the loop term slightly decreases the $X$ position of the saddle point and increases $\eta$, breaking the scaling symmetry. From $X \approx 10$ and below, we see that $\eta$ is no longer small enough for slow roll.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$X_{\text{saddle}}$</th>
<th>$\eta$</th>
<th>$L = 0$</th>
<th>$X_{\text{saddle}}$</th>
<th>$\eta$</th>
<th>$L = 2.33$</th>
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<td>1</td>
<td>123.2163</td>
<td>−0.0060</td>
<td>123.2163</td>
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</table>

Table 1: A table of how $\eta$ changes when we include the $\alpha'$ corrections and move the minimum to smaller $X$ by changing $\lambda$.

It could be possible that having broken the scaling symmetry we now just abandon it and fine tune the other parameters again to find a new way to get inflation out of these regions. However, this is not the approach we should take. $A, B$ should be set by the gaugino condensation. In particular they depend on the complex structure moduli which should also be fixed by the fluxes. We should think of these as pinned down by the theory and then we should ask the sorts of questions we have done here. In particular we would like to use the requirements of inflation to constrain the parameter space available to us, in particular the size of the compact manifold. Also the size of the gauge groups that make the condensate should be more "natural". A product group such as $SU(100) \times SU(90)$ is not very likely (quantify this and perhaps constrain with the 18 from brustein). So in fact we should be looking at smaller more realistic groups which as we see from equation() will place the turning points at smaller $X$. It is for this reason that we think that a study of the small radius or large coupling behaviour could be important to our understanding of this mechanism.
8 Comments on more Corrections

8.1 $g_s$ Corrections

8.2 Validity of the Two-stage process

The keen critic may worry that in our analysis we have applied the "algorithm" set out by KKLT outside of its region of validity. Specifically, in the KKLT scenario we use the Ramond-Ramon and Neveu-Schwartz fluxes to fix the complex structure moduli and the dilaton and then by adding non-perturbative effects we manage to fix the Kähler modulus at large volume. This advocates a two stage process that may not necessarily hold. Surely we should fix all the moduli at once rather than in stages as is advocated for by [de Alwis]. Remember, the non-perturbative corrections depend on $g_s$ so really the two steps are not decoupled. However, one might believe that it is at least feasibly to perform such a two stage process so long as the mass scales involved are widely separated. The mass scale for fixing the complex structure moduli and the dilaton with fluxes is expected to be around the string scale whereas the mass scale for the nonperturbative effects is much smaller as these terms are $e^{-aT}$ and $T$ is assumed to be large. Since the mass scales for the effects responsible for fixing these moduli are so separated we expect it to be feasible to stabilise these moduli independently of each other.

However, in this work we are probing the region of smaller volume and one might expect that by doing so we raise the mass scale of the non-perturbative terms so much that we can not transport the two stage fixing process of KKLT. One might even question if our result is a reflection of applying this process outside of the region of expected validity. We can see that this is simply not the case by looking at the nonperturbative corrections $e^{-aT}$ and realising that even at the smallest volumes we get to is very small. Specifically, the smallest volume we probe is really an intermediate volume of $T \times 10$ in Planck units. Even at this volume the non-perturbative terms are at largest $2^{-10} \ll 1$. One should understand from this that the SUGRA analysis we have done remains the same as KKLT and RI and at intermediate volumes the results are robust. Nonetheless, if one would like to perform a cosmological analysis from the potentials arising from such models, the lesson is that we need to be very careful as the results are very sensitive to the corrections that may arise. In particular, we see that even at intermediate values of the volume, $\alpha'$ corrections are sufficient to prevent inflation from occurring in a region of parameter space where we might have expected inflation to occur.

In fact in studying how the $\alpha'$ corrections may destabilise the results of RI, we looked both at the possibility that the corrections would destabilise the minimum for the volume and that the corrections would ruin the conditions for inflation. We found the latter occurring and in fact in the region where one expects $\alpha'$ corrections to be sufficient we find that the minimum always survives. This hints to the fact that at these scales the SUGRA analysis is robust to $\alpha'$ corrections whereas a cosmological analysis is particularly sensitive to them. In general the landscape appears to be a hostile place for inflation. (too tongue in cheek?)

From this we have learnt that even at a scale where one can still believe all of our SUGRA analysis one cannot always trust a cosmology analysis.

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$^2$We thank Alex Saltman for useful discussions on this point
9 Acknowledgements

Pfister, NASA, etc

10 Calculating $\alpha'$ corrections

In principle, rather than probing the effect of the $\alpha'$ loop corrections, we could calculate the exact Kähler potential using mirror symmetry. Since we are working with $N = 1$ compactifications with fluxes we can no longer trust such calculations and it would perhaps be better to specifically calculate the loop corrections for a specific Calabi-Yau and look at the effects of this term. This is the approach we take, and we will also comment on why we think it is sufficient to neglect the instanton corrections.

In this work we study the quintic three-fold $\mathbb{P}_4(5)$, with $b_{11} = 1$ Kähler moduli and $b_{21} = 101$ complex structure moduli and Euler number $\chi = -200$. To calculate the loop correction to the Kähler potential we need to know the behaviour of the prepotential $\mathcal{F}(w)$ which is a function of $b_{11}+1$ homogeneous coordinates $w^j$. In our case $b_{11} = 1$, so we have $w^1, w^2$.

The full prepotential is calculated [Greene, candelas] in terms of $t \equiv \frac{w^1}{w^2}$

$$\mathcal{F}(w) = (w^2)^2 \left\{- \frac{5}{6} t^3 - \frac{11}{4} t^2 + \frac{25}{12} t - \frac{25}{2\pi^3} \zeta(3) + \text{exponentially small} \right\}$$

There are terms that are cubic, quadratic and linear in $t$, which correspond to the bare prepotential. The term independent of $t$ is the loop term and the exponentially small terms correspond to instanton corrections. In fact the loop term is proportional to the Euler number $\chi$. I have just absorbed this here. It may turn out to be interesting later. We will not discuss the instanton corrections here except to say that they are terms $\sim e^{2\pi i k t}$ where $k$ is an even integer. We say they are small because we are interested in the contributions of the imaginary part of $t$. In the range of values we will study these contributions turn out to be much smaller than the loop correction so we believe this to be a consistent choice. Specifically the instanton corrections will be at largest $O(10^{-60})$ compared to the loop correction term which turns out to be $O(1)$.

Now that we have $\mathcal{F}(w)$ we can calculate the Kähler potential using

$$K = -\log \left\{ i \left( \frac{\partial \mathcal{F}(w)}{\partial w} - w^j \frac{\partial \mathcal{F}(w)}{\partial w^j} \right) \right\}$$

Explicitly substituting in $\mathcal{F}(w)$ and ignoring the exponentially small instanton corrections we find that only the cubic (bare term) and the constant (loop) term contribute to $K$. Note that after differentiation you can set $w^2 = 1$ as it is projective coordinate and we find

$$K = -\log \left( \frac{5}{6} \left( \frac{t - \bar{t}}{i} \right)^3 + \frac{50}{\pi^3} \zeta(3) \right)$$

Removing the factor of $5/6$ from in front of the $\left( \frac{t - \bar{t}}{i} \right)^3$ term to compare to the usual results for large $\text{Im}(t)$ we find

$$K = -\log \left( \left( \frac{t - \bar{t}}{i} \right)^3 + \frac{60}{\pi^3} \zeta(3) \right) + \log \left( \frac{6}{5} \right)$$
The extra constant term in $K$ would simply rescale $V$. Since this does not affect the metric as it depends only on derivatives we can drop the extra constant dependence. Hence, we consider

$$K = -\log \left( \left( \frac{t - \bar{t}}{i} \right)^3 + \frac{60}{\pi^3} \zeta(3) \right)$$

which is now in a form we can use. The large $\text{Im}(t) = \frac{t + \bar{t}}{2\pi}$ limit agrees with the large radius result usually quoted

$$K = -3 \log \left( \frac{t - \bar{t}}{i} \right)$$

Since we want to use the language of many papers where $W \sim e^{-aT}$ and $K = -3 \log(T + \bar{T})$ we change notations by noting that $T(t) = -it$ and the Kähler potential becomes

$$K = -3 \log(T + \bar{T})$$

Including the loop term we have

$$K = -\log \left( (T + \bar{T})^3 + L \right)$$

where $L = \frac{60}{\pi^3} \zeta(3) \approx 2.33$. We will use the language of $T$ rather than $t$ henceforth to make comparisons with other work simple.
References


[7] Ross, Sarkar ..... 


[12] GKP....

[13]