DC and RC Circuits

Introduction
This lab is intended to demonstrate simple concepts that you learned in 1402 and provide you some experience with simple electrical circuits and measurements of the properties of these circuits. You will hopefully gain some practical intuition on the nature of the electrical potential and will experimentally verify Ohm’s law and the rules for equivalent resistance and capacitance of multi-resistor or multi-capacitor circuits. You will also directly study the properties of RC circuits and verifying the relationship between resistance, capacitance and the time-constant of these circuits. The equipment that you will be using for this experiment is quite modern and maybe even overly sophisticated for our purposes. One secondary goal of this lab is that you become sufficiently familiar with the operation of the brand new oscilloscopes that you will be able to effectively use them for the AC circuits lab.

Background Material
In 1402 you learned that the potential energy of a particle with charge $q$ in an electric field can be written, $U(x) = qV(x)$, where $V$ is the position-dependent electric potential associated with the electric field. You saw that when a particle moves along a path from a to b, the change in electric potential is given by

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$$

The negative sign reflects conservation of energy and has the immediate consequence that the electric potential decreases (increases) along paths which are parallel (anti-parallel) to the electric field. Another critically important lesson that you learned in 1402 was that because electrostatic forces are “conservative”, the change in potential is independent of the path taken from a to b. You also learned that while ideal conductors cannot have non-zero electric fields in the conductor, “real” conductors can have electric fields and that these fields induce bulk motion of charge in the form of electric currents. The nature of such conductors is that the current density in the conductor is proportional to the applied electric field, $J = \sigma E$. This relationship, which is valid at any point in the conductor, can be re-expressed for conductors of constant cross-sectional area $A$ in terms of the “resistance” of the entire conductor defined $R = |\Delta V|/I$. $I$ is the “current” in the conductor, $I = JA$, and the resistance is given by $R = \rho L/A$. Alternatively, you could also define $R$ operationally as the ratio of the voltage drop across a piece of conductor carrying current $I$, to the current itself:

$$R = \frac{|\Delta V|}{I}.$$  

In principle, $R$ defined this way could depend on the current, which would imply a violation of Ohm’s law. You should not be surprised that such violations might occur in real-world materials, however these deviations are sufficiently small to be of no consequence for most practical applications.

Equivalent Resistance
You also learned in 1402, rules for calculating the equivalent resistance of combinations of resistors. We will be demonstrating these rules in this lab and it’s worth reminding you how they arise. Figure 1 shows resistors in parallel and in series. Remember that when resistors are in series, by definition they both carry the same current. The change in electric potential across the two resistors in the sum of the changes across each resistor, $\Delta V = -R_1I - R_2I = -(R_1 + R_2)I$. So, if we use the above operational definition of $R$, we get for the resistance of the pair, $R = R_1 + R_2$. 
For resistors in parallel, the same potential difference $\Delta V$ is applied across both resistors. This is essentially the definition of what it means for resistors to be in parallel. The total current flowing through the two resistors is, then, the sum of the currents flowing through each resistor:

$$I = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

The equivalent resistance of the combination is then given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Remember that when resistors are combined in series, the equivalent resistance is always larger than either individual resistance so adding a resistor in series in a circuit always reduces the current flowing through that part of the circuit. However, when you add a resistor in parallel in a circuit you provide another path through which current can flow. Thus, the equivalent resistance is always reduced and the current flowing through the combined resistors always increases. From here on out we will drop the explicit $\Delta$ in $\Delta V$ and also drop the explicit sign associated with the decrease in the electric potential across a resistor and write $V=IR$, the form of Ohm’s law that you’re likely most familiar with.

**Capacitors**

In 1402 you learned that combinations of conductors can store charge if a potential difference is applied across the conductors. The potential difference requires an electric field and the electric field produces charge at the surface of the conductors. Operationally, one can define the “capacitance” of a capacitor as the ratio of charge stored to applied potential difference,

$$C = \frac{Q}{\Delta V}.$$  

For parallel plates with no dielectric shown in Fig. 2, you saw in 1402 that the capacitance is given by $\varepsilon_0 A/d$. As with resistors, we will drop the explicit $\Delta$ and write $C=Q/V$ or $Q=CV$.

Remember that equal and opposite charges are stored on the plates of a capacitor so the $Q$ is the magnitude of the charge stored on one plate. As with resistors, it is possible to derive rules for the equivalent capacitance of combinations of capacitors. Refer to Fig. 3. When capacitors are connected in parallel, they each have the same potential difference $V$. Thus, the total charge stored on the pair of capacitors is

$$Q = C_1 V + C_2 V = (C_1 + C_2) V.$$  

Then, using the above definition of capacitance we get an equivalent capacitance of the pair

$$C = \frac{Q}{V} = C_1 + C_2.$$  

However, when capacitors are connected in series, two two capacitors are required to store the same charge. This is illustrated in Fig. 3. The low potential conductor of the first capacitor and
the high potential conductor of the second capacitor are effectively isolated from the rest of the
circuit and together must remain electrically neutral. So, $Q_1 = Q_2$. Then, the potential drop across
the two capacitors is

$$V = V_1 + V_2 = \left( \frac{Q}{C_1} + \frac{Q}{C_2} \right) = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

So, the equivalent capacitance of the pair is given by

$$\frac{1}{C} = \frac{V}{Q} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)$$

RC Circuits

Let’s look at the behavior of capacitors in electrical circuits. If the applied potential difference
across the capacitor is constant with time, the charge stored on the capacitor is also constant. The
rate of change of the charge on one of the capacitor plates (e.g. Fig. 3) is given by $I = \frac{dQ}{dt}$.
Thus, if the charge on the capacitor is constant no current flows onto or off the conductors of the
capacitors. In such “static” situations, the capacitor plays no role in electrical circuits since it
looks like an “open” or infinite resistance connection. On the other hand, if the voltage across the
capacitor changes with time, then, by definition, so does the charge and current will flow onto
and off the two conductors of the capacitor. Because the charges on each conductor are equal add
opposite, the currents on each plate are opposite sign and equal in magnitude. Thus, it looks like
the current associated with the charging or discharging of the capacitor in response to change
applied voltages simply passes through the capacitor as shown in Fig. 2.

Now let’s look at what happens when we make a simple circuit with a capacitor, resistor, power
supply and switch as shown in Fig. 4. We will label all voltage and current values for the current
and resistor lowercase for this problem because they are time dependent. Initially the switch is
open and no charge is stored on the capacitor. At $t=0$ we move the switch to position A and see
what happens in the circuit. Initially, there’s no charge in the capacitor so the voltage across the
capacitor $v_0 = 0$. Thus, the voltage produced by the power supply $V$ is applied all across the
resistor, so a current $i_0 = V/R$ flows around the circuit. But, because there is current flowing
“through” the capacitor, the capacitor is charging (remember that what is really happening is that
positive charge is flowing onto the high-potential side of the capacitor and negative charge is
flowing onto the low-potential side). If we require the net change in potential around the closed
loop to be zero, we get the equation

$V - iR - q/C = 0.$

We’ll take the derivative of each term and plug in $I = dq/dt$, to get

$R \frac{di}{dt} = -i/C.$

Let’s integrate this over time applying the initial condition $i_0 = V/R$,

$R \int_{V/R}^{i} \frac{di'}{i'} = - \frac{1}{C} \int_{0}^{t} dt' \rightarrow \ln i|_{V/R} = - \frac{1}{RC} t \rightarrow \ln i - \ln(V/R) = \ln(i/(V/R)) = - \frac{1}{RC} t,$

where $t'$ and $i'$ are dummy variables of integration. If we exponentiate both sides we get

$i = \frac{V}{R} e^{-t/RC}.$

The charge on the capacitor as a function of time can be obtained by integrating this equation,

$q = \int_{0}^{t} \frac{V}{R} e^{-t'/RC} dt = \frac{V}{R} \left( -RC \right) e^{-t'/RC} \bigg|_{0}^{t} = VC \left( 1 - e^{-t/RC} \right).$

At long times the exponential term is zero and the charge on the capacitor approaches the steady-
state value of $CV$ which is what would be expected if the potential difference provided by the
power supply is applied directly across the capacitor. This makes sense, however, because when
the charge is constant, no current can flow through the capacitor which also means that no current
can flow through the resistor which means that there is no voltage drop across the resistor. Thus,
the full potential difference provided by the power supply is indeed applied across the capacitor. The above equation for the current as a function of time directly shows you that the current exponentially decreases to zero at long times. The factor $RC$ in the above equations necessarily has units of time (since the argument to the exponential must be a pure number) and we call this quantity the “time constant” of the circuits. Large time-constants imply that a circuit takes a long time to reach its static state, short time-constants indicate that the circuit reaches its static state quickly.

A similar analysis to that performed above can be performed for a discharging capacitor. Suppose we start with a fully charged capacitor to a potential difference $V_0$ and at $t=0$ throw the switch in Fig. 5 to position B. Prove to yourself that the time dependence of the charge and current in the resistor is given by,

$$q = V_0 e^{-t/RC}, \quad I = \frac{V_0}{R} \left(1 - e^{-t/RC}\right)$$

Include the demonstration of these results in your lab write-up.

**Lab Equipment**

The main pieces of apparatus that you will be using in this laboratory are:

- “Bread-board”
- Multi-meters (2).
- Function generator.
- Oscilloscope.

You will also be using individual resistors and capacitors to build small circuits, pieces of “jumper” wire to make connections in your circuit, and cables with “banana” plugs and/or “alligator clips” to measure voltages and currents in your circuit.

![Figure 5. Rough schematic of your breadboard.](image)
The breadboard is the most essential piece of equipment as it allows you to build simple circuits and provides all power that you will need. A partial and simplified sketch of the breadboard is given in Fig. 5. The breadboard provides small sockets into which you can plug the ends of resistors, capacitors, jumper wires etc. All of the sockets in a row are electrically shorted so you connect two components together on the breadboard by plugging the two components into any two (or more) sockets on the same row. The breadboards that you will be using have rows at the top which provide DC power and a reference ground. By connecting one end of a jumper wire to one of the power rows and the other end of the jumper wire to a row in the main part of the breadboard you can provide power to your circuits. You ground your circuit by providing a similar connection between the low-potential side of the circuit and the ground strip at the top.

The second most critical pieces of equipment that you’ll be using are the multi-meters. Your setups include two such meters, a hand-held meter and a desk-top meter. Many of the measurements that you’ll be making are most easily made with the hand-held meter, but current measurements, which require the meter to be in the circuit, are most easily made with the desk-top meter and cable connections to the bread-board. Both meters require you to manually set the range of the meter and switch between voltage/resistance and current settings. Here are some things to keep in mind when using the multi-meters:

- To avoid confusion, always connect the “black” cable on the hand-held meter to the ground/“common” terminal on the meter and the red cable to the V/Ω/I terminal.
- Be very careful not to “short” the circuit by inadvertently touching the meter probes to components that are at different voltages. Place your components on the breadboard with the leads well separated to minimize the chances of such shorts.
- You should never need to use the 10 A terminal on the multi-meters. When measuring currents use the 2 A terminal only.
- When measuring voltage differences, always use the “black”/common/ground cable on the low-potential side of the circuit/component. This way you will always get the right sign for the potential difference.
- When measuring resistances, make sure to keep a steady contact of the multi-meter leads to both ends of the resistor and make sure that the measurement is stable before accepting the values.
- Never try to measure the resistance of a resistor while it’s connected in a circuit. If you do so, you will be measuring the effective resistance of the entire circuit. Remove it first, measure the resistance and then put it back into the circuit.
- Make sure you know what units the multi-meter is measuring in. The desk-top units in particular use different units on different ranges (e.g. mV vs V).

![Square-wave pulse train](image_url)
The oscilloscope is the most sophisticated piece of equipment that you will be using this semester. The scopes are brand-new desk-top digital oscilloscopes of size and performance that would have been unthinkable a few years ago. The purpose of the oscilloscope is basically to allow you to visually observe the time variation of voltages on one or two input. The scope starts from a “trigger” and plots the voltage as a function of time over a range determined by you and over a voltage scale also determined by you. The instructions for the lab below will walk you through the use of the oscilloscope. The oscilloscope uses special coaxial cables that provide the scope with both the signal and a “ground”. You should see a short cable coming off the probe with an alligator clip at the end. If not, ask your TA to help.

The function generator provides signals of a pre-determined shape, magnitude, and frequency for various uses. We will be using the square-wave generator that provides a sequence of “square-wave” pulses like that shown in Fig. 6. The period \( T \) of the pulses is given by the time from the start of one rising edge to the start of the next rising edge. The generator that you will be using provides a square-wave of 50% “duty cycle” which means that the voltage is at the nominal amplitude for half the period. The frequency, as always, is the inverse of the period, \( f = 1/T \). The amplitude \( V_{\text{max}} \) is the height of the pulses in volts.

You will be using a variety of resistors, and you’ll need to be able to read their marking in case you mix the different values up. Resistors are marked with a sequence of colored rings that indicate the value and tolerance of the resistor. The color codes for the values are

- **black** – 0, **brown** – 1, **red** – 2, **orange** – 3, **yellow** – 4, **green** – 5, **blue** – 6, **violet** – 7, **grey** – 8, **white** – 9.

Resistors are marked in a two significant digit exponential notation, \( xy \times 10^z \), where x, y, and z are the 1\(^{\text{st}}\), 2\(^{\text{nd}}\), and 3\(^{\text{rd}}\) rings respectively. For example a 100\(\Omega\) resistor is marked brown, black, brown \((10\times10^1)\) as shown above. A fourth ring marks the tolerance with the most common values, silver – 10%, gold - 5%, brown – 1%. You can find a clever Java-based resistor color-decoder at: [http://www.broadcast.net/resistor.html](http://www.broadcast.net/resistor.html). It won’t be useful to you during the lab but if you want to familiarize yourself with reading resistor values before the lab check it out.

**Laboratory Procedures**

In this lab we will start with very simple measurements and procedures to make you comfortable with using the apparatus and work you up to more complicated tests and measurements culminating in the measurement of time-constants of RC circuits using the oscilloscope.
Part 1 – DC circuits

1. Start by measuring and recording the actual voltage provided by the 5V supply on your breadboard with the multi-meter. You will be using this supply for most of your lab so you will need to be sure that you know the true potential difference that it provides relative to the ground on the breadboard. You can use the terminals provided at the top right corner of the breadboard to make this measurement. Make sure the multi-meter is set to measure voltage (voltage/resistance for the hand-held meter) and that it’s set to the correct scale (20 V maximum).

2. Select a 1kΩ resistor and measure its resistance with the multi-meter. Now connect the 5V supply to a row of the breadboard with a jumper wire. **Make all of your connections with the breadboard turned off and then turn it on when you are done.** This prevents problems from inadvertent shorts as you assemble your circuit. Plug one lead of the resistor into another socket on the same row and the other end into a separate row on the breadboard. It will be easiest if you pre-bend the leads on the resistor so that they are perpendicular to the body of the resistor before trying to insert it into the breadboard. Finally jumper the unconnected end of the resistor back to the ground strip with another jumper wire. You have now made the simple DC circuit shown in Fig. 8a. Measure the voltage drop across the two ends of the resistor. If you have properly connected it to the power supply and ground you should nominally find the voltage you measured in step 1.

3. Now disconnect the low-potential side of the resistor from ground. You will now use the desk-top multi-meter to measure the current flowing through the resistor. Use a cable with an alligator clip on one end to connect the current (A) terminal of the multi-meter to your circuit. You can either clip onto the low-potential lead of the resistor or connect with a short piece of jumper wire open the same row of the breadboard. Similarly connect the common/ground terminal of the multi-meter to the ground of the breadboard (either directly or with a jumper wire). You should now have a complete circuit through the multi-meter. Switch the multi-meter to current measurement and set the range appropriately. Measure the current flowing through your resistor. Compare to what you expect given the measured power supply voltage and the resistance of your resistor. Once you have measured the current, disconnect your circuit from the 5V supply and ground leaving everything else connected. Measure the complete resistance of your circuit including jumper wires and multi-meter (on the same current setting). How is this value different from just the value of the resistor itself?

4. In this step we will make a simple voltage divider circuit that we will also use in later steps.

![Figure 8. DC Circuits that you will make in Part I of the lab, steps 1-5.](image-url)
For this part we will use 2 1kΩ resistors so select another and carefully measure its resistance. Connect them in series according to the circuit shown in Fig. 8b. Keep track of which resistor is which since you will need to know the values in later steps. Measure the current through the two resistors as you did in part 3. Now, measure the voltage relative to ground at the middle connection between the two resistors with the hand-held meter. Again be careful not to short the resistors with the lead of the multi-meter. It’s easiest if you touch the ground lead of the multi-meter to the ground terminal on the breadboard. In your write-up analyze the circuit and explain the value you get for the resistance at this point.

5. Now we will observe what happens with resistors connected in parallel. Connect a 100 Ω resistor in parallel with the second of the 1kΩ resistors as shown in Fig. 8c. You can now measure the current flowing in the circuit by measuring the voltage drop across the first resistor since all current passes through that resistor. You’ll need to use the actual measured resistance, of course. You should see that the current has increased substantially. In fact, the value you get should be close to \( \frac{5V}{(1kΩ + 100Ω)} = 4.5mA \). Why ? Re-measure the voltage at the mid-point of the circuit again (marked \( V' \) in figure 8c). Analyze the circuit and quantitatively demonstrate in your write-up why you obtained your measured values.

6. The voltage divider circuit you made in part 4 is of general use in electronic circuits because it allows you to “step down” a supplied voltage to a lower value in a well-defined way. The term “divider” is used because the value at the mid-point between the two series resistors is given in terms of ratios of resistances. The “output” voltage of the divider is the mid-point value and this voltage can be used to power another circuit in some application. However, you saw in part 5, that the voltage at the mid-point can change dramatically depending on the “load” resistance of that other circuit (represented by the 100Ω resistor in step 5). We will study that behavior in more detail here using the variable resistor provided at the bottom of your breadboard. You should see at the bottom a short socket row adjacent to a knob labeled 1K-10KΩ and marked with the “slashed” resistor symbol shown in Fig. 9. The markings show you that there are two sockets for each side of the resistor. Jumper the variable resistor into your circuit in place of the 100 Ω resistor and also connect two short jumpers with their ends in the air so you can measure and adjust the resistance. Measure the “output” voltage of the divider and the current flowing through the circuit (using voltage drop across first resistor) for “load” resistances of 1k, 2k, 3k, 4k, 5k, 6k, 7k, 8k, 9k, 10 kΩ (obtained by tuning the variable resistor). You should see that for large load resistances, the “output” voltage is close to 2.5 V, but as you lower the load resistance the “output” voltage from the divider starts to “sag”. Analyze the circuit and explain quantitatively how this works. Compare your
measurements for “output” voltage and current flow with what you expect from the analysis of your circuit.

Part II. RC Circuits
In this part of the lab you will use the square-wave produced by the function generator to stimulate the charging and dis-charging of capacitors in a circuit and measure the time-constants of the circuit. You will use these measurements to demonstrate the rules for equivalent capacitance of capacitors in series and in parallel.

1. Disassemble your circuits from part I. Connect a 10kΩ resistor and a 0.01 µF capacitor in series as shown in your bread board in Fig. 10a. **Measure the resistance of the 10kΩ resistor first.** Now connect the function generator across both the resistor and capacitor. Your function generator probably has a piece of wire on the ground terminal that you can connect directly to the low-potential side of the capacitor. The signal side of the function generator can be connected to the resistor with an alligator clip. Set the function generator to square-wave output and select the frequency range of 1 Khz.

2. We are now going to observe the output of the function generator on the oscilloscope. Connect one scope probe to channel 1 of the oscilloscope. Hook the end of the signal probe to the signal generator output, the resistor to which it’s connected or a short piece of jumper wire connected in the same row of the breadboard which ever is most convenient. Clip the ground reference lead on the probe to the ground of your circuit by the most convenient method. If you haven’t done so already, turn on the oscilloscope via the switch on top of the unit. It will go through a self-test. The operation of the scope is controlled completely via on-screen menus. There is a “menu” button on the face which will scroll through different menus. For each menu, there are “action” buttons on the side of the scope which change a particular setting. Chances are you may already see the square wave on the scope, but we first want to make sure the scope is set up properly.

- **Trigger:** Use the menu button to scroll through the menus until you find the “trigger” menu. You should set the trigger channel to channel 1, the trigger coupling to “AC” coupling and set the scope to trigger on the rising edge of the signal.
- **Inputs:** again scroll through the menus until you find the “inputs” menu. For now you want channel 1 on and channel 2 off. You want channel 1 to be DC coupled.
- **Adjust trigger:** turn the trigger level knob on the right hand side of the scope back and forth until you see the square wave appear on the screen. What you are doing is adjusting the threshold of the input signal (on channel 1) at which the scope starts to plot the input voltage. Since you are providing a square pulse you should have no trouble making the scope trigger.

By now if you’ve connected the scope properly and made the above settings you are probably seeing your square wave on the scope. If not, ask your TA for help.

You will now adjust the scope scale settings to the values you need. The voltage and time scales are shown on the screen on these scopes. The values are the potential difference and time range corresponding to one “division” (or square) on the screen. You will want something like 0.5-1V per vertical division and 100 µs per horizontal division. You can adjust the scope settings with the voltage scale adjust knob above the input for channel 1 and the time-base adjust knob on the right hand side of the scope. There are vertical and horizontal adjustment knobs on the scope that let you move the trace around for your convenience. Make sure these are nominally centered – otherwise you may have trouble
finding your signal. You may also find that if you change the vertical scale on the input the scope will stop triggering. If so tweak the trigger level and it should start triggering again.

3. Now we need to adjust the function generator. Adjust the amplitude of the function generator to 1V (for convenience) and adjust the frequency so that the period is exactly 1 ms. If you have set the scope to 100 μs per division that means you should see the start of two rising edges 10 divisions apart.

4. Now we want to measure the voltage across the capacitor. Connect another scope probe to channel 2 of the oscilloscope and connect the two leads of the probe across the capacitor. It’s easiest if you simply connect to the leads of the capacitor. Find the “input” menu of the scope again, turn on channel 2 and set it to DC coupling. Now you should see two signals being plotted – your square wave and a “rounded” square wave. The rounding is the exponential charging and dis-charging of the capacitor as the potential difference provided by the function generator turns on and off.

5. In this step your goal is to measure the RC time-constant of the exponential charging and
discharging of the capacitor. When the capacitor is charging, the time-constant is determined by when the voltage reaches the value $V_{\text{max}}(1-e^{-1}) = 0.63\ V_{\text{max}}$. When the capacitor is discharging, the time-constant is given by the time it takes for the voltage to drop to $V_{\text{max}}e^{-1} = 0.37\ V_{\text{max}}$. You will use the cursor functions of the scope to perform these measurements. Refer to the illustration in Fig. 11. Adjust the vertical position of the trace so that it is on the boundary of a division – preferably at the middle of the screen. Set one cursor at “zero” i.e. at the output of the function generator when it is “off” and adjust the other cursor until the $\Delta V$ reading is $0.63\ V$ (you must have properly adjusted the function generator output). Now set one horizontal cursor at the rising edge of the square wave and place the other such that it crosses the signal trace exactly where the upper cursor crosses. The $\Delta t$ reading on the scope should be the time-constant of the circuit. How does the value compare to what you expect? Verify that you get the same result for the dis-charging of the capacitor.

6. Move the scope probe on channel 2 so that you are measuring the voltage across the resistor. Now you are effectively measuring the current flowing through the circuit via the voltage drop across the resistor. Sketch what you see in your lab book. In your write-up explain why the current behaves as you observe.

7. Now that you can measure RC time-constants you will verify the rules for equivalent capacitance of series and parallel capacitors. Add a second $0.01\ \mu F$ capacitor in series with the resistor and capacitor as shown in Fig. 10b. Now measure the time-constant of the circuit using the voltage drop provided by the current flow through the resistor. Do not get confused about the voltage level at which you measure the time-constant. Use the same falling/rising method given above. Now put the second capacitor in parallel with the first as shown in Fig. 10c and re-measure the time constant. In your write-up extract the equivalent capacitances of the circuits and compare to what you expect given the rules for equivalent capacitances.