1. Decays of the spin–1/2 baryons

Most of the spin–1/2 baryons in the “baryon octet” (nucleon, Λ, Σ, Ξ) decay weakly, to another spin–1/2 baryon plus a pion. The two exceptions are the Σ⁰ (which decays electromagnetically) and the neutron (which decays weakly to $pe^-\bar{\nu}_e$).

- Show that none of the particles in the octet can decay strongly.
- Show that the Σ⁰ is the only particle in the octet that can decay electromagnetically.
- Explain the unusual decay pattern for the neutron.

2. Consequences of isospin

Suppose the strong interaction Hamiltonian is invariant under an $SU(2)$ isospin symmetry, $[H_{\text{strong}}, I] = 0$. By inserting suitable isospin raising and lowering operators $I_\pm = I_1 \pm iI_2$ show that (up to possible phases)

$$
\frac{1}{\sqrt{3}} \langle \Delta^+ | H_{\text{strong}} | \pi^+ p \rangle = \frac{1}{\sqrt{2}} \langle \Delta^+ | H_{\text{strong}} | \pi^0 p \rangle = \langle \Delta^0 | H_{\text{strong}} | \pi^- p \rangle.
$$

3. Decay of the Ξ⁺

The Ξ⁺ baryon decays primarily to Ξ + π. For a neutral Ξ⁺ there are two possible decays:

$$
\Xi^{+0} \rightarrow \Xi^0\pi^0
$$

$$
\Xi^{+0} \rightarrow \Xi^0\pi^+
$$

Use isospin to predict the branching ratios.
4. $\Delta I = 1/2$ rule

The $\Lambda$ baryon decays weakly to a nucleon plus a pion. The Hamiltonian responsible for the decay is

$$H = \frac{1}{\sqrt{2}} G_F \bar{u} \gamma^\mu (1 - \gamma^5) d \bar{s} \gamma_\mu (1 - \gamma^5) u + c.c.$$ 

This operator changes the strangeness by $\pm 1$ and the $z$ component of isospin by $\mp 1/2$. It can be decomposed $H = H_{3/2} + H_{1/2}$ into pieces which carry total isospin $3/2$ and $1/2$, since $\bar{u} \gamma^\mu (1 - \gamma^5) d$ transforms as $|1, -1\rangle$ and $\bar{s} \gamma_\mu (1 - \gamma^5) u$ transforms as $|1/2, 1/2\rangle$. The (theoretically somewhat mysterious) “$\Delta I = 1/2$ rule” states that the $I = 1/2$ part of the Hamiltonian dominates.

(i) Use the $\Delta I = 1/2$ rule to relate the matrix elements $\langle p \pi^- | H | \Lambda \rangle$ and $\langle n \pi^0 | H | \Lambda \rangle$.

(ii) Predict the corresponding branching ratios for $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$.

The PDG gives the branching ratios $\Lambda \to p\pi^- = 63.9\%$ and $\Lambda \to n\pi^0 = 35.8\%$. 