1. Equations of motion

(i) Derive the Euler-Lagrange equations of motion by varying the fields in

\[ L = \frac{1}{2} \dot{x}^2 + i \overline{\psi} \dot{\psi} - \frac{1}{2} (W')^2 + \overline{\psi} \psi W''. \]

Vary the fields in the usual way,

\[ x \rightarrow x + \delta x \quad \psi \rightarrow \psi + \delta \psi \quad \overline{\psi} \rightarrow \overline{\psi} + \delta \overline{\psi}, \]

but remember that \( \psi, \delta \psi, \overline{\psi}, \delta \overline{\psi} \) all anticommute.

(ii) Use the (anti-) commutation relations

\[ i [p, x] = 1 \quad \{ \psi, \overline{\psi} \} = 1 \]

to derive the Heisenberg equations of motion for the Hamiltonian

\[ H = \frac{1}{2} p^2 + \frac{1}{2} (W')^2 - \frac{1}{2} (\overline{\psi} \psi - \psi \overline{\psi}) W''. \]

Assuming your results match with part (i), this shows we quantized our fermions correctly.

2. Supercharges from Noether’s theorem

(i) Show that the Lagrangian

\[ L = \frac{1}{2} \dot{x}^2 + i \overline{\psi} \dot{\psi} - \frac{1}{2} (W')^2 + \overline{\psi} \psi W'' \]

changes by a total time derivative under

\[ \delta x = \xi \psi + \overline{\psi} \xi \]
\[ \delta \psi = -i \xi (\dot{x} + i W') \]
\[ \delta \overline{\psi} = i \xi (\dot{x} - i W') \]

(ii) Use Noether’s theorem to find the conserved charges \( Q, \overline{Q} \) associated with this symmetry.
3. Component expansion of the action

Work out the expansion of the action

\[ S = \int dt d^2 \theta \left( -\frac{1}{2} \bar{D}FDF - W(F) \right) \]

in terms of the component fields \( x, \psi, \bar{\psi}, d \).