Michelson and Fabry Perot Interferometers and Study of Channeled Spectra

References

1. ** Jenkins and White. “Fundamentals of Optics” (any edition). This has excellent figures and description (Chapter on Interference of Two Beams of Light - also use index). (Some Xerox copies of this part may be available on loan.)

2. Any other text on Physical Optics giving a detailed description of the Michelson Interferometer, its use, and the fringe patterns for different conditions, including white light fringes.

3. Rainwater. Mimeograph Optics Notes on Physical Optics (a few copies are available for loan).

Preparation - Read one of the references (preferably including the discussion in Jenkins and White). Read the theory of the measurement of the mica thickness by both methods in these instructions.

Apparatus - Interferometers

The Michelson and Fabry Perot Interferometers are Adam Hilger Ltd. instruments where one mirror can be moved over a few cm distance using a drive handle in front, which gives 2 mm motion per turn. A coarse scale on the left gives readings to the nearest mm. The drive handle also turns a rotary scale (above the handle), which goes 0-100 twice per turn, so each division is 0.01 mm, permitting position readings to the nearest 0.01 mm. On the right there is a fine motion dial reading 0-100 divisions. When engaged, this causes 0.01 mm motion per turn, or 0.0001 mm per dial division. The engagement lever is down for disengaged, up for engaged. However, care must be taken when moving it up beyond the half up position to see that the threads engage properly. The dial should be turned slowly as the lever is raised beyond mid-height. If the threads are not engaged properly, a binding effect is noted for the dial rotation, which is not present if the threads engage properly. If there is binding, lower the lever and turn the dial a few turns before trying again. Note that the main turning handle should not be forced (used) unless the fine motion is disengaged. The lever should always be lowered when finished, or when the fine motion is not being used.

The motion should be limited to the region between 10 and 20 mm on the left scale to prevent damage. (For the M. I.).

The Mirrors: DO NOT TOUCH ANY OF THE OPTICAL SURFACES.
If you think that a surface needs cleaning, consult the instructor in charge. The half silvered film is smoky in appearance, and may be mistaken for dirt. Even the lightest touch of lens paper will deface a partially coated mirror if the film happens to be of silver. The fully coated surfaces are without protection and may be marred by a touch of a finger or any hard object.

Rough adjustment of the fully coated surfaces is accomplished by three thumb screws at the back of the mirrors \( M \) and \( M' \) of the Michelson Interferometer. If the adjustment is sufficiently close to see the fringes (for a setting near 15 mm), use the fine motion adjustments only for the subsequent adjustment. The Fabry-Perot only has adjustments for the far mirror (which is the moveable one). Use only the fine adjust for it (F-P), and do not decrease the spacing enough to cause the mirrors to touch (readings equal to or greater than zero on the Fabry-Perot are allowed).

**PART ONE** (First Week)

The following exercises are intended to gain familiarity with the manipulations of the Michelson Interferometer. The initial relative mirror tilts are probably such that Hg green (\( \lambda = 5460.7\text{Å} \)) fringes can be seen when the Hg arc source is turned on and the green filter is in place at the source. If the \( \sim 3'' \) diameter focusing lens is not in position, the illuminated solid angle of view is just the small value of the source viewed from \( \sim 30 \) inches. Note that when the focusing lens is properly positioned between the source and the interferometer, the larger illuminated solid angle is that of the lens aperture viewed from a shorter distance.

1. **Qualitative Study of Monochromatic Light**

Starting near a 12 mm scale reading, note the fringe pattern. If the image of the right mirror is parallel with the back mirror, you will see a circular fringe pattern. Also, as your eye moves in a left to right, back and forth motion, the pattern will remain relatively unchanged, vs. having a source (new rings emerging from the pattern center) or sink (rings vanishing into the center) effect. The horizontal fine motion control on the right mirror should be used to attain minimum (horizontal eye motion) source-sink effect. The other fine motion control is used to minimize the vertical (eye oscillatory motion) source-sink effect.

When maximum parallelism has been obtained, start at 10 mm (left scale) and note the pattern. For effectively parallel mirrors spaced \( d \) apart, the path length difference for the two paths is \( 2d(1-\theta^2/2) \), where \( \theta \) is the ray angle with the mirror normal. One goes from one fringe to the next when this changes by \( \lambda \), or \( \lambda = 2d(\theta \Delta \theta) \). The “angular tightness” of the fringe pattern is thus proportional to \( d \), and the number of circular fringes increases in proportion to \( (d\theta^2) \).
Moving the coarse handle in ¼ turn steps, note the “tightness” of the fringe pattern for each setting. The changes should first increase the fringe spacing until \( d \approx 0 \), where a uniform field ideally results. Subsequently, \( d \) increases and the fringes become more closely spaced. Study the interval 10 to 20 mm. The reading (to the nearest 0.01 mm) should be recorded for the most “open” pattern (fewest fringes).

Repeat turning the fine horizontal tilt screw for the right mirror ~1 turn, so that curved, mainly vertical, fringes result (i.e., circular with the center displaced to the left or right). In this case, the effect of passing through \( d = 0 \) is to reverse the curvature of the fringes (note the position where this happens) when going from 10 to 20 mm as before.

White Light Fringes

There is a white light source of variable intensity with a diffusing screen. Place it in front of the Hg source and adjust for a white light field. The mirror tilt adjustment should be such that ~4 vertical fringes are seen for the Hg green line when near the 13 mm position. Colored fringes are seen using white light in a small positioning region near \( d = 0 \). The condition is that the difference in “optical path lengths” (distance times index of refraction) is near zero over the range \( \lambda \sim 5000 \) to 6500 Å where most of the visible white light is concentrated. The pattern is discussed in “Jenkins and White”. In our case, the symmetry center is a white, rather than a dark, fringe (since \( u \approx 0 \) for the real part of the silver index of refraction).

Start with a position setting ~½ turn short of the \( d = 0 \) position as found in (1). Engage the fine motion dial (cautiously) and study the pattern as you (quickly) turn the dial in ~½ turn steps (releasing between). The effect is an unmistakable “rainbow” effect when it appears. You will probably overshoot on the first search. Have the instructor show you how to back up a little, overcoming the large “backlash” lag that would otherwise result, so you can slowly bring the pattern into view. Use the telescope to view the fringe patterns with magnification. Note the color order and magnitude as one goes away from the pattern center about which there is essential reflection symmetry for the pattern. (In the report, explain why the colors are as seen.) Note the distance to the nearest 0.0001 mm for reference.

4. Measuring \( \Delta \lambda \) of the Hg Yellow Lines.

Replace the green filter with the yellow filter that only passes the two Hg yellow lines near 5769.7 Å and 5790.7 Å, which are spaced 21.065 Å. The two lines are of essentially equal intensity.

Measure the wavelength difference between the two yellow mercury lines by finding the distance between successive positions of the movable mirror at which the visibility of the fringes is at a minimum.

When radiation of two different wavelengths is present in an interferometer source, two different fringe systems will be formed. As the path difference is changed,
these two fringe systems will move at different rates because of their different wavelengths. In the Michelson Interferometer, the width of a bright fringe is equal to the width of an adjacent dark fringe. For this reason, the two fringe systems will not appear as distinct fringe systems. At positions of the mirror, however, where the bright fringes of one set coincide with the bright fringes of the other set, there will be the strongest contrast between the bright and dark fringes, or maximum visibility. At position of the movable mirror where the bright fringes, due to light of one wavelength, coincide with the dark fringes of the other set, the contrast between light and dark fringes will be least, and there will be minimum visibility. The condition of minimum visibility is more readily recognized than that of maximum visibility and should be used for the following measurements.

Let D be the distance the carriage moves between two successive positions of minimum visibility. The path difference changes by 2D. This path difference change, however, will include one more fringe for the shorter wavelength, \( \lambda_2 \), than for the longer, \( \lambda_1 \), or,

\[
m \lambda_1 = (m+1) \lambda_2 = 2D,
\]

so

\[
m (\lambda_1 - \lambda_2) = \lambda_2 \quad \text{and} \quad m = \frac{2D}{\lambda_1}
\]

Eliminating \( m \), and solving for \( \lambda_1 - \lambda_2 \),

\[
(\lambda_1 - \lambda_2) = \frac{\lambda_1 \lambda_2}{2D} \approx \frac{\lambda^2}{2D}, \quad \text{if} \quad \lambda_1 \text{and} \lambda_2 \text{are nearly equal},
\]

where \( \lambda = (\lambda_1 + \lambda_2)/2 \) is the average. Thus, if 2D is measured, and relatively rough values of \( \lambda_1 \) and \( \lambda_2 \) (or a mean value) are known, the wavelength difference may be calculated with precision (better than 1% accuracy). Readings should be made to 0.0001 mm.

It will be found impossible to determine accurately the carriage position for any one minimum of visibility. Satisfactory precision may be obtained, however, by starting with the carriage 2 mm before the white light fringe setting. The dial position settings should be recorded for a starting (0th) minimum (visibility), and at the 1st, 2nd, . . . , 9th, 10th, subsequent minima. Then go the 20th, 30th, 40th, . . .. recording every tenth minimum. More accuracy is obtained this way. A total distance \(~4\) mm should be covered. Repeat once.

Note: While the screw motion gives a monotonic change of mirror position with increasing dial readings, the motion of such devices tends to have some non-linearity for motions less than integer numbers of full turns of the main screw. One of the reasons for carrying the measurements over \(>40\) times the spacing of successive visibility minima is to have a total motion \(>2\) mm. Using the 0, 1, 2 . . . . . . 10 positions, it should be easy to see if you miscounted and subsequently correct the readings (i.e., changing through 9 or
11 visibility minima rather than 10 will be evident in the analysis and the count can be corrected.

When measuring distances, always move the carriage away from the half silvered mirror, i.e. in the direction of increasing scale readings. (Note the relation between the motions.) (There is “play” in drive so a reverse motion has delayed start to take up “slack”)

In the analysis, prepare a table having columns for \( n \), the position reading, \( y_n \) (to 0.0001 mm), and the differences \( D_n = (y_n - y_{n-1}) \). \( D_n \) for \( n = 1 \) to 10 will have a mean, \( <D> \), and a standard deviation value, \( S_D \). The distances corresponding to \( n = 10 \) to 20, 20 to 30, and 30 to 40 should be approximately 10 times \( <D> \) for \( n = 1 \) to 10. It will be obvious if the \( n \) differences are not = 10 (such as 8, 9, 11, 12) and the \( n \) values should be corrected if this is the case. Now use the calculator’s best straight line fit, \( y_n = a + bn \), to evaluate \( b \) and \( S_b / b \). Note that \( b \) is the best \( D \) to evaluate \( \Delta \lambda = (\lambda_n - \lambda_{n-1}) \) and its fractional uncertainty is just \( (S_b / b) \). Compare with the known value and discuss the comparison.
PART TWO (First Week)

Measurement of the thickness of a mica sheet using the Michelson Interferometer and using the method of channeled spectra. (Use Mica #1 sample.)

A. Theory for measurement using the Michelson Interferometer

In a two beam interferometer such as the Michelson interferometer, the wave front is split into two parts, which follow separate paths, which are later recombined. The two path lengths can be different, since one path length is adjustable. For the mirrors near enough to “parallel” and using strictly monochromatic light of wave length $\lambda$, a fringe pattern is always seen, with maxima where the relative phases differ by zero or an integer number of cycles, and minima where an odd half integer number of cycles are involved for the phase difference $\delta$. If one could slowly vary $\lambda$ for fixed interferometer settings, the fringe pattern would shift as $\lambda$ is changed. For a broad line of width $\Delta \lambda$ such that changing $\lambda$ by $\Delta \lambda$ makes the fringe shift by $\geq 1$ fringe separation, the patterns from the different $\lambda$ in the $\Delta \lambda$ interval tend to give a blur with no fringes visible. However, if $(d\delta / d\lambda)\Delta \lambda << 1$ cycle, the fringe patterns coincide and a net fringe system is visible. For white light fringes, the phase difference $(d\delta / d\lambda)\Delta \lambda = (\delta_2 - \delta_1)$ for the two paths must remain essentially constant over the range of $\lambda$ producing the visible effect. Usually $\lambda \sim 5000$ to 6000 A.U. covers the main region of visible effect. For equal path lengths through the glass pieces, this occurs only for equal air paths, $\delta = 0 = (\delta_2 - \delta_1)$.

When a thickness $t$ of mica is placed in one path (for a double traversal), the extra optical path length in units of $\lambda$ is $2t(\mu - 1) / \lambda$, where $\mu$ varies with $\lambda$. To achieve white light fringes, the mirror must be moved closer by distance $d$, such that $2t(\mu - 1 - 2d) / \lambda$ is (essentially) independent of $\lambda$. A choice of $\lambda_A \sim 5780$ AU, $\lambda_B \sim 5461$ AU brackets the main $\lambda$ region contributing to the visual effect. The condition that the above expression is nearly independent of $\lambda$, is

$$
\frac{\mu_A t - (d_1 + t)}{\lambda_A} = \frac{\mu_B t - (d_1 + t)}{\lambda_B},
$$
giving:

$$
t = \frac{d_1(\lambda_A - \lambda_B)}{\lambda_A(\mu_A - 1) - \lambda_B(\mu_A - 1)}
$$

For the kind of mica used here, it has been established that the values of $\mu$ at the indicated $\lambda_1$, $\lambda_2$, $\lambda_3$, are the tabulated $\mu_1$, $\mu_2$, $\mu_3$: 
\( \lambda (\text{Å}) \) | \( \mu \)  
---------- | -------- | -------- | ---------------  
\( \lambda_A \rightarrow \lambda_1 \) | \( \mu_1 \) | 1.5939  
\( \lambda_B \rightarrow \lambda_2 \) | \( \mu_2 \) | 1.5959  
\( \lambda_3 \) | \( \mu_3 \) | 1.6061  

It is suitable to use \( \mu = 1.5959 + 0.01862 \left( \frac{5461 \text{Å}}{\lambda} \right)^2 - 1 \) which is easily calculated.

**Procedure:**

The mica (use #1) is on a mount which permits it to be placed in front of the movable mirror so the mica covers the lower half of field of view. The position of the white light fringes through mica comes before the position of the fringes in air. The idea is to measure the distance \( d \) required to shift the pattern from the center of the white light fringes in the mica to the white light fringes on the air side. Use the telescope with its pointer to view the pattern for more precise final settings. Since the linearity of the threads is not of desired accuracy over small fractions of the 2 mm motion, the final measurement counts the motion in units of green line mercury fringes (use vertical fringes).

A. Without the telescope, establish the position settings for the white light fringes in mica and then in air. Record and repeat a few times to check reproducibility. (Knowing these values helps in part B.)

B. Move the telescope into position and focus on the pointer and (as best you can) on the fringes. The pointer should be near the center of the field of view defined by the interferometer mirrors (using Hg green illumination). Now, with white illumination, find at what mirror positioning the center of the white light fringe pattern is on the pointer (through the mica). (The previous reading helps for this setting).

The suggested procedure is as follows: first set so the center of the white light fringe pattern (through the mica) is on the pointer (telescope). Remove the white light source so the green pattern (air side) is seen. Note the fraction (nearest tenth) of the fringe spacing to move a bright fringe as to center it on the pointer. Turn the fine knob clockwise counting the fringes as they pass the pointer. The total distance is between 150 and 180 fringes shift. At 150, restore the white source to see if you are near. Increase with the green lines (white source removed) in steps of two fringes, checking with the white source after each shift. The idea is to judge the distance to the center of the white light fringe pattern in air to \( \sim 0.1 \) fringe, if better than \( \sim 0.5\% \) accuracy is to be achieved. Repeat a few times to be sure of the integer part of the count. Calculate \( t \) using \( d \) from counting green fringes and also from the scale reading changes (compare).

Second Week
1. This week you should use the Fabry-Perot interferometer to similarly measure the spacing of the two yellow sodium D lines ($\lambda = 5889.950$ Å and $\lambda = 5895.924$ Å). The Hilger motion controls of the movable mirror are exactly the same as for the Michelson interferometer. The fringe pattern is much sharper, however, so use best overlap of the two fringe patterns rather than the (inapplicable) minimum visibility condition. The two mirrors should always be made parallel so the fringes appear to be at $\infty$. The touching condition of the mirrors is to be avoided. It occurs near (-0.7mm). Thus, you can start near zero on the scale. Note each best overlap position over a 4 mm range and repeat. (Usually, results better than 1% are obtained.)

Using a procedure similar to the one for determining $\Delta \lambda$ of the Hg yellow lines with the M.I., calculate $\Delta \lambda$ from the best straight line fit $b$ and the fractional uncertainty $\frac{\lambda}{S_b} = \frac{b}{b}$.

Note: As the Na lamp warms up, each line pressure broadens, with a “self-reversal” absorption (by the cooler Na vapor near the wall). This causes each line to look like a close double for $d \geq 3\text{mm}$. At best overlap, the net effect is that one of the double lines overlap.

B. Theory for Channeled Spectra

A semi-reflecting transmitting mirror is placed at 45° with the collimator (as indicated). A white light filament lamp illuminates the mirror through the lens, to give white light incident normally on the mica. The mica, lens and light source positions are adjusted to give an image of the filament reflected from the mica on the slit, for maximum brightness of the spectrum. The slit is narrowed to yield alternating bright and dark bands. The dark bands are at $\lambda$ values where the light reflected from the front and rear mica surfaces have an integer number of cycles different, $(2\mu t / \lambda) = n$. If a dark band occurs at $\lambda_A$ and another at $\lambda_B < \lambda_A$, separated $m$ dark bands away, then, if the optical path $(2\mu_A t / \lambda_A)$ is $n$ cycles different for $\lambda_A$, for $\lambda_B$ the optical path $(2\mu_B t / \lambda_B)$ is $(n + m)$ cycles different, so that:

$$\frac{2\mu_A t}{\lambda_A} = n, \quad \frac{2\mu_B t}{\lambda_B} = (n + m), \quad \text{so} \quad m = 2t \left( \frac{\lambda_B - \lambda_A}{\lambda_A} \right)$$

or

$$t = \frac{m\lambda_A \lambda_B}{2[\mu_B \lambda_A - \mu_A \lambda_B]}$$

Use this relation to evaluate $t$.

The change in $\mu$ with $\lambda$ is important. If it were neglected, using $\mu_A \approx \mu_B \approx \mu_A$, an error of many percent would be introduced.

Procedure: First, the monochromator dial readings are calibrated using a mercury source by noting the dial indications for the mercury lines (compare with values in the attached table). The clearly visible lines are the main lines and five red lines beyond
6000Å. Make a plot of \((\lambda_{\text{true}} - \lambda_{\text{scale}}) = y\) vs. \(\lambda_{\text{scale}} = x\) and draw a smooth “best eye fit” curve to obtain the correction that must be applied to the \(\lambda_{\text{scale}}\) readings (below).

For measuring the thickness of the mica, starting as far as possible in the red where dark bands can be suitably located, note the position \(\lambda_0\) of some initial dark band \((m = 0)\). Then move in steps of 10 bands at a time, recording \(\lambda_{10}, \lambda_{20}, \ldots, \lambda_{100}\) for \(m = 10, 20, 30, \ldots, 100\).

For the analysis, prepare a table having columns for \(m (= x)\) and \(y = \left[\frac{\mu}{\lambda} - \frac{\mu_0}{\lambda_0}\right]\). Use the formula for the index of refraction of mica given in the discussion of the Michelson Interferometer. A best straight line fit \(y = a + bm\) has \(b = 1/(2t)\), or \(t = 1/(2b)\). (Show that this is true.) First, evaluate the differences between \(y\) values for adjacent \(\Delta m = 10\) values of \(n\). The comparison of these differences provides a check of \(\Delta m = 10\) in each case, since \(\Delta m = 8, 9, 11,\) or \(12\) (etc.) would show up in comparing these values. Make corrections of the \(m\) values if so indicated and re-calculate the table. It is also interesting to calculate \(t\) for each \(\Delta m = 10\) interval and for each \(y_m\) value. Finally, make a best straight line fit for all values of \(m\) and \(y_m\). Determine \(b\) and \(S_b / b\) to determine \(t\) and the fractional uncertainty \((S_b / b)\). It is best to express \(t\) in microns. Repeat the measurements and the analysis two more times and compare these results with those using the M.I. and discuss.

Repeat for a thinner mica (mica #2) using \(\Delta m = 1\) or 2 steps from the red to green where minima are measurable.
Mercury Optical Spectral Lines in the Visible Region “seen” in the EKA Hg Arc Sources.

An “A” in front denotes that the value is from the A.I.P. Handbook (Second Edition, p.7-121, or Third Edition, p.94). The term scheme is shown on p.7-38 (2nd. Ed.) or p.7-24 (3rd Ed.). The initial and final transition states are given, as are the relative intensities (visual), indicated by the letters s, m, and w, which stand respectively for strong, medium, and weak, and v, which stands for very. Some weaker lines, B, not given in A, are taken from the MIT wavelength tables. All of these lines were seen using our Bausch and Lomb Prism Monochromator.

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